Network Systems in Science and Technology

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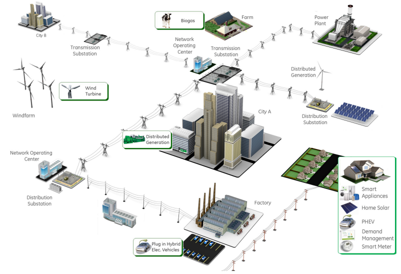
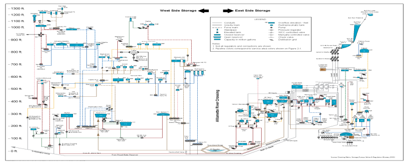
Network systems in technology



Smart grid

Amazon robotic warehouse



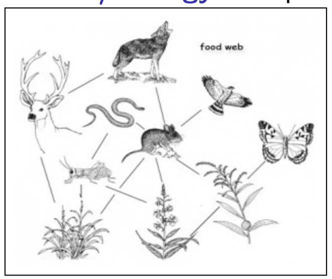
Portland water network

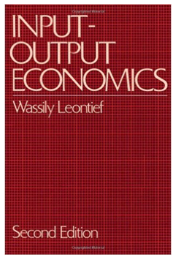
Industrial chemical plant

Network systems in sciences

Sociology: opinion dynamics, propagation of information, performance of teams

Ecology: ecosystems and foodwebs
Economics: input-output models
Medicine/Biology: compartmental systems





Acknowledgments

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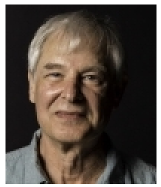
Waterloo



Noah Friedkin
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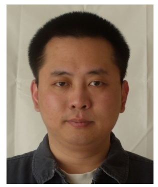


Peng Jia
UCSB

NSF AFOSR ARO ONR DOE

Outline

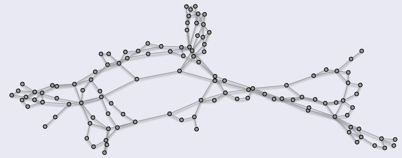
1. Intro to Network Systems
   Models, behaviors, tools, and applications

2. Power Flow
   "Synchronization in oscillator networks" by Dörfler et al, PNAS '13
   "Voltage collapse in grids" by Simpson-Porco et al, NatureComm '16

3. Social Influence
   "Opinion dynamics and social power" by Jia et al, SIREV '15

Linear network systems

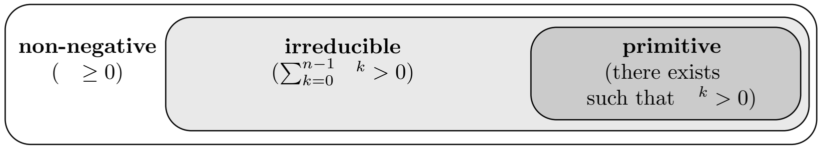
$$x(k+1) = Ax(k) + b \qquad \text{or} \qquad \dot{x}(t) = Ax(t) + b$$



1. systems of interest
2. asymptotic behavior
3. tools

network structure $\Longleftrightarrow$ function = asymptotic behavior

Perron-Frobenius theory



if A **non-negative**

1. eigenvalue $\lambda \geq |\mu|$ for all other eigenvalues μ
2. right and left eigenvectors $v_{\text{right}} \geq 0$ and $v_{\text{left}} \geq 0$

if A **irreducible**

3. $\lambda > 0$ and λ is simple
4. $v_{\text{right}} > 0$ and $v_{\text{left}} > 0$ are unique

if A **primitive**

5. $\lambda > |\mu|$ for all other eigenvalues μ
6. $\lim_{k\to\infty} A^k/\lambda^k = v_{\text{right}} v_{\text{left}}^T$, with normalization $v_{\text{right}}^T v_{\text{left}} = 1$

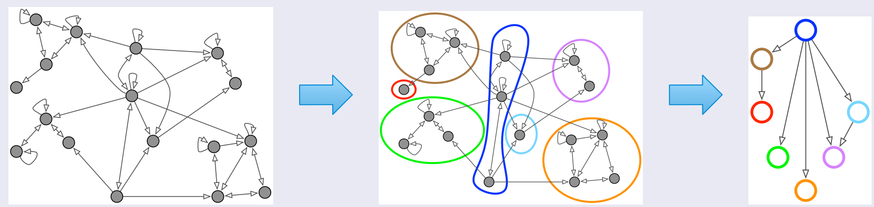
Algebraic graph theory

Powers of $A \sim$ paths in G:

$(A^k)_{ij} > 0$ $\Longleftrightarrow$ there exists directed path of length k from i to j in G

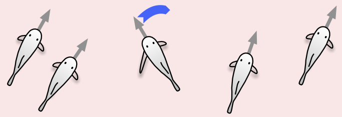
Primitivity of $A \sim$ paths in G:

A is primitive ($A \geq 0$ and $A^k > 0$) $\Longleftrightarrow$ G strongly connected and aperiodic (exists path between any two nodes) and (exists no k dividing each cycle length)



digraph — strongly connected components — **condensation**

Averaging systems



Swarming via averaging

$$x_i^+ := \text{average}\big(x_i, \{x_j, j \text{ is neighbor of } i\}\big)$$

$$x(k+1) = Ax(k)$$

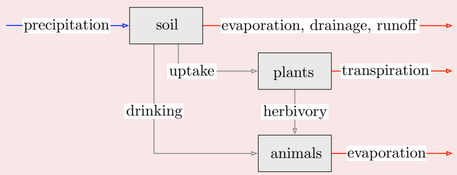
A **influence matrix**:
row-stochastic: non-negative and row-sums equal to 1

For general G with multiple condensed sinks
(assuming each condensed sink is aperiodic)

⇒ consensus at sinks
convex combinations elsewhere

consensus: $\lim_{k\to\infty} x(k) = \big(v_{\text{left}} \cdot x(0)\big)\mathbb{1}_n$
where v_{left} = convex combination = influence centrality

Compartmental flow systems



Water flow model for a desert ecosystem

$$\dot q_i = \sum_j (F_{j\to i} - F_{i\to j}) - F_{i\to 0} + u_i$$

$F_{i\to j} = f_{ij} q_i, \quad F = [f_{ij}]$

$$\dot q = \underbrace{\big(F^T - \text{diag}(F\mathbb{1}_n + f_0)\big)}_{=:C} q + u$$

C **compartmental matrix**:
quasi-positive (off-diag ≥ 0), $f_0 \geq 0 \implies$ weakly diag dominant
Analysis tools: *PF for quasi-positive, inverse positivity, algebraic graph*

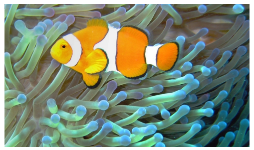
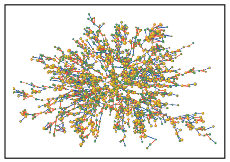
system (= each condensed sink) is outflow-connected ⇔ C is Hurwitz

⇒ $\lim_{t\to\infty} q(t) = -C^{-1}u \geq 0$
$(-C^{-1}u)_i > 0 \iff i$th compartment is inflow-connected

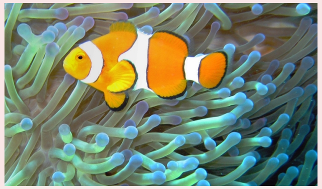
Nonlinear network systems

Rich variety of emerging behaviors

1. equilibria / limit cycles / extinction in populations dynamics
2. epidemic outbreaks in spreading processes
3. synchrony / anti-synchrony in coupled oscillators


Population systems in ecology



Mutualism between clownfish and anemones

Lotka-Volterra: x_i = quantity/density

$$\frac{\dot x_i}{x_i} = b_i + \sum_j a_{ij} x_j$$

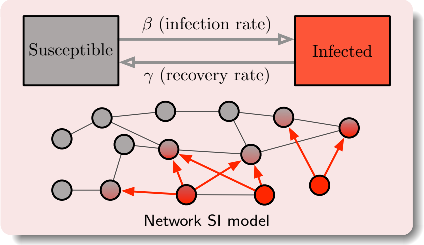
$$\dot x = \text{diag}(x)(Ax + b)$$

A **interaction matrix**:
$(+,+)$ mutualism, $(+,-)$ predation, $(-,-)$ competition
rich behavior: persistence, extinction, equilibria, periodic orbits, ...

1. **logistic growth:** $b_i > 0$ and $a_{ii} < 0$
2. **bounded resources:** A Hurwitz (e.g., irreducible and neg diag dom)
3. **mutualism:** $a_{ij} \geq 0$

⇒ exists unique steady state $-A^{-1}b > 0$
$\lim_{t\to\infty} x(t) = -A^{-1}b$ from all $x(0) > 0$

Network propagation in epidemiology



Network SIS: (x_i = infected fraction)

$$\dot{x}_i = \beta \sum_j a_{ij}(1 - x_i)x_j - \gamma x_i$$

(rescaling)

$$\dot{x} = \big(I_n - \mathrm{diag}(x)\big)Ax - x$$

A **contact matrix**: irreducible with dominant pair $(\lambda, v_{\mathrm{right}})$

below the threshold: $\lambda < 1$

- 0 is unique stable equilibrium
- $v_{\mathrm{right}}^T x(t) \to 0$ monotonically & exponentially

above the threshold: $\lambda > 1$

- 0 is unstable equilibrium
- unique other equilibrium $x^* > 0$
- $\lim_{t\to\infty} x(t) = x^*$ from all $x(0) \neq 0$

Analysis methods

1. **nonlinear stability theory**
2. **passivity**
3. **cooperative/competitive system and monotone generalizations**

Mutualistic Lotka-Volterra: $\dot{x} = \mathrm{diag}(x)\big(Ax + b\big)$

A quasi-positive and Hurwitz $\implies$ inverse positivity

cooperative systems theory: (if Jacobian is quasi-positive, then almost all bounded trajectories converge to an equlibrium)

Network SIS: $\dot{x} = \big(I_n - \mathrm{diag}(x)\big)Ax - x$

A irreducible, above the threshold $\lambda > 1$

monotonic iterations and LaSalle invariance

Incomplete references on linear network systems

Averaging: multi-sink, concise proofs, etc

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- D. G. Luenberger. Introduction to Dynamic Systems: Theory, Models, and Applications. John Wiley & Sons, 1979.

Incomplete references on nonlinear network systems

Lotka-Volterra models

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- Y. Takeuchi. Global Dynamical Properties of Lotka-Volterra Systems. World Scientific, 1996.
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Network SI/SIS/SIR models

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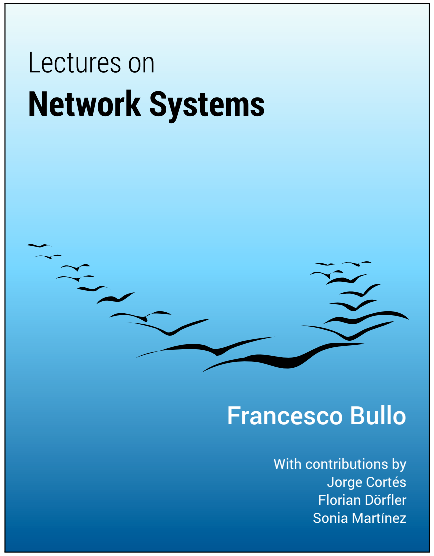
New free text "Lectures on Network Systems"



Lectures on Network Systems, v. .85
For students: free PDF for download
For instructors: slides and answer keys

Linear Systems:

1. motivating examplesss from social, sensor and compartmental networks,
2. matrix and graph theory, with an emphasis on Perron–Frobenius theory and algebraic graph theory,
3. averaging algorithms in discrete and continuous time, described by static and time-varying matrices, and
4. positive and compartmental systems, described by Metzler matrices.

Nonlinear Systems:

5. formation control problems for robotic networks,
6. coupled oscillators, with an emphasis on the Kuramoto model and models of power networks, and
7. virus propagation models, including lumped and network models as well as stochastic and deterministic models, and
8. population dynamic models in multi-species systems.

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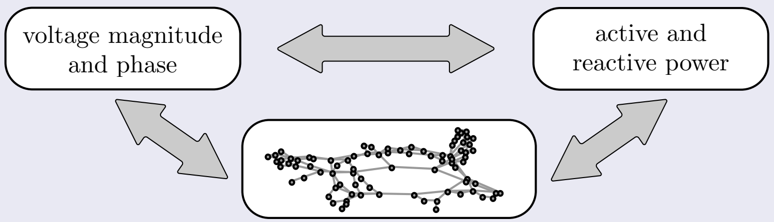
Power flow equations

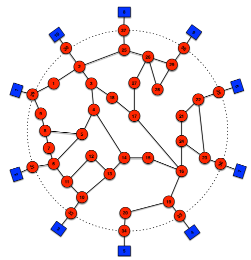


1. secure operating conditions
2. feedback control
3. economic optimization

while accurate numerical solvers in current use,
much ongoing research on optimization,
network structure $\Longleftrightarrow$ **function = power transmission**

Power networks as quasi-synchronous AC circuits

1. **generators** ■ and **loads** ●
2. **physics:** Kirchoff and Ohm laws
3. today's simplifying assumptions:
   1. **quasi-sync:** voltage and phase V_i, θ_i
      active and reactive power P_i, Q_i
   2. lossless lines
   3. approximated decoupled equations



Decoupled power flow equations

$$\begin{aligned} \text{active:} \quad P_i &= \textstyle\sum_j a_{ij} \sin(\theta_i - \theta_j) \\ \text{reactive:} \quad Q_i &= -\textstyle\sum_j b_{ij} V_i V_j \end{aligned}$$

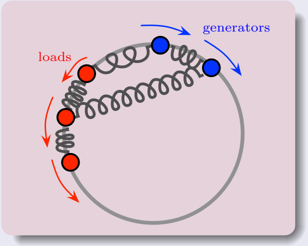
Power Flow Equilibria

$$P_i = \sum_j a_{ij} \sin(\theta_i - \theta_j) \qquad Q_i = -\sum_j b_{ij} V_i V_j$$

As function of network structure/parameters

1. do equations admit solutions / operating points?
2. how much active / reactive power can network transmit?
3. how to quantify stability margins?

Active power dynamics and mechanical analogy



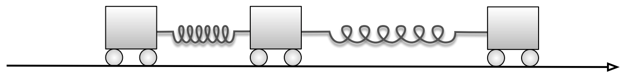
Coupled swing equations

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j a_{ij} \sin(\theta_i - \theta_j)$$

Kuramoto coupled oscillators

$$\dot{\theta}_i = P_i - \sum_j a_{ij} \sin(\theta_i - \theta_j)$$

Lessons from linear spring networks



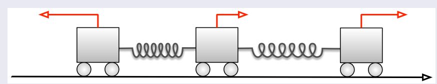
Force $\propto$ displacement:

$$F_i = \sum a_{ij}(x_j - x_i) = -(Lx)_i$$

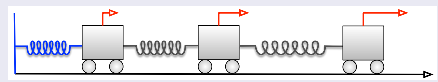
Laplacian / stiffness matrix and connectivity strength:

$$L = \mathrm{diag}(A \mathbb{1}_n) - A$$

$$\lambda_2 = \text{second smallest eigenvalue of } L$$

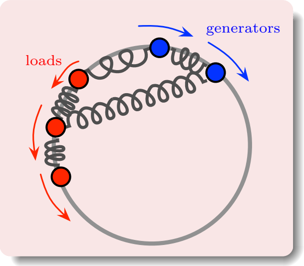


$$x = L^{\dagger} F_{\text{load}}$$



$$x - x_{\text{rest}} = L^{-1}_{\text{grounded}} F_{\text{load}}$$

Active power / frequency equilibrium conditions



Given balanced P, do angles exist?

$$P_i = \sum_j a_{ij} \sin(\theta_i - \theta_j)$$

connectivity strength vs. **power transmission**:

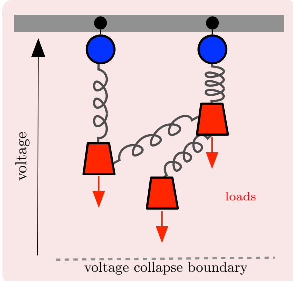
#1: "torques" $\sim$ active powers P_i
"displacements" $\sim$ power angles $(\theta_i - \theta_j)$

#2: with **increasing power transmission**,
$(\theta_i - \theta_j)$ approach $\pi/2$ = **sync loss**

Equilibrium angles (neighbors within $\pi/2$ arc) exist if

$\lVert \text{pairwise differences of } P \rVert_2 < \lambda_2(L)$	for all graphs
$\lVert \text{pairwise differences of } L^{\dagger} P \rVert_\infty < 1$	for trees, 3/4-cycles, complete

Reactive power / voltage equilibrium condition



Given reactive Q_{loads}, do voltages V_{loads} exist?

$$Q_i = -V_i \sum_j b_{ij}(V_j - V_{\text{rest},j})$$

where V_{rest} = open-circuit voltages

connectivity strength vs. **power transmission**:

#1: "force" $\sim$ reactive load Q_{loads}
"displacement" $\sim$ relative voltage variation

#2: with **increasing inductive Q_{loads}**,
V_{loads} falls until **voltage collapse**

Equilibrium voltage (high-voltage solution) exist if

$$\left\lVert L^{-1}_{\text{grounded,scaled}} Q_{\text{loads}} \right\rVert_\infty < 1$$

Summary (Power Flow)

New physical insight

1. sharp sufficient conditions for equilibria
2. upper bounds on transmission capacity
3. stability margins as notions of distance from bifurcations

Applications

1. secure operating conditions:
   realistic IEEE testbeds (Dörfler et al, PNAS '13)
2. feedback control:
   microgrid design (Simpson-Porco et al, TIE '15)
3. economic optimization:
   convex voltage support (Todescato et al, CDC '15)

Incomplete references on power flow equations

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- A. Araposthatis, S. Sastry, P. Varaiya Analysis of power-flow equation. *Int. Journal of Electrical Power & Energy Systems*, 3, 1981.
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Our recent work

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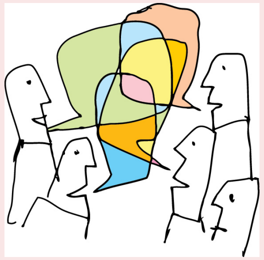
Social power along issue sequences

- **Deliberative groups in social organization**
  - government: juries, panels, committees
  - corporations: board of directors
  - universities: faculty meetings
- **Natural social processes along sequences**:
  - levels of openness and closure?
  - influence accorded to others? emergence of leaders?
  - rational/irrational decision making?

Groupthink = "deterioration of mental efficiency . . . from in-group pressures," by I. Janis, 1972

Wisdom of crowds = "group aggregation of information results in better decisions than individual's" by J. Surowiecki, 2005

Opinion dynamics and social power along issue sequences



DeGroot opinion formation

$$y(k+1) = Ay(k)$$

Dominant eigenvector v_{left} is **social power**:

$$\lim_{k\to\infty} y(k) = (v_{\text{left}} \cdot y(0))\mathbb{1}_n$$

- $A_{ii} =: x_i$ are **self-weights / self-appraisal**
- A_{ij} for $i \neq j$ are **interpersonal accorded weights**
- assume $A_{ij} =: (1 - x_i)W_{ij}$ for constant W_{ij}

$$A(x) = \text{diag}(x) + \text{diag}(\mathbb{1}_n - x)W$$

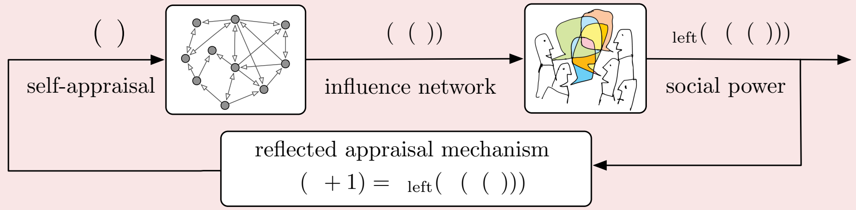
- $w_{\text{left}} = (w_1, \dots, w_n) =$ dominant eigenvector for W

Opinion dynamics and social power along issue sequences

Reflected appraisal phenomenon (Cooley 1902 and Friedkin 2012)

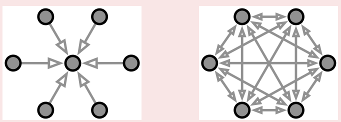
along issues $s = 1, 2, \dots,$ individual dampens/elevates self-weight according to prior influence centrality

self-weights ⇐ relative control on prior issues = social power



⇒ $$v_{\text{left}}(A(x)) = \Big(\frac{w_1}{1-x_1}, \dots, \frac{w_n}{1-x_n}\Big) \Big/ \sum_{i=1}^{n} \frac{w_i}{1-x_i}$$

Influence centrality and power accumulation



Existence and stability of equilibria?
Role of network structure and parameters?
Emergence of *autocracy* and *democracy*?

For strongly connected W and non-trivial initial conditions

1. **convergence to unique fixed point (= forgets initial condition)**

$$\lim_{s\to\infty} x(s) = \lim_{s\to\infty} v_{\text{left}}(x(s)) = x^*$$

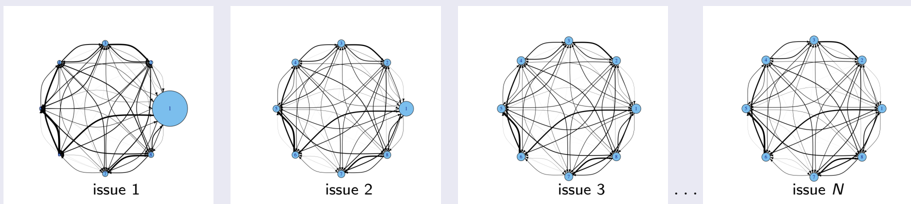
2. **accumulation of social power and self-appraisal**
   - fixed point $x^* = x^*(w_{\text{left}}) > 0$ has same ordering of w_{left}
   - social power threshold p: $x_i^* \geq w_i \geq p$ and $x_i^* \leq w_i \leq p$

Emergence of democracy

If W is doubly-stochastic:

1. the non-trivial fixed point is $\frac{\mathbb{1}_n}{n}$
2. $\lim_{s\to\infty} x(s) = \lim_{s\to\infty} v_{\text{left}}(x(s)) = \frac{\mathbb{1}_n}{n}$

- Uniform social power
- No power accumulation = evolution to democracy

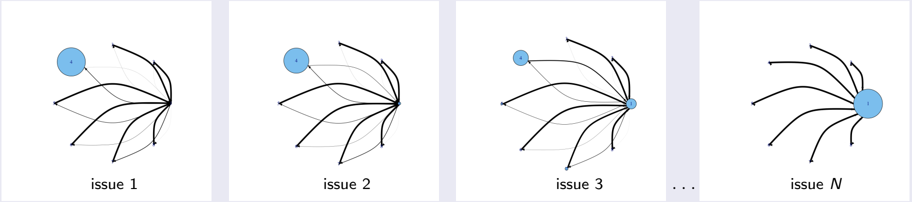


Emergence of autocracy

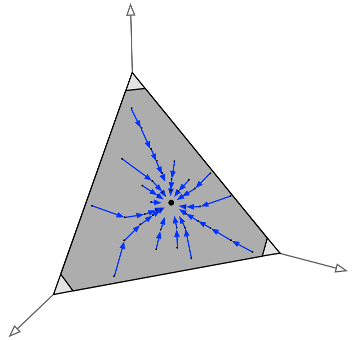
If W has star topology with center j:

1. there are no non-trivial fixed points
2. $\lim_{s\to\infty} x(s) = \lim_{s\to\infty} v_{\text{left}}(x(s)) = \mathbb{e}_j$

- Autocrat appears in center node of star topology
- Extreme power accumulation = evolution to autocracy



Analysis methods



1. existence of x^* via **Brower fixed point theorem**
2. **monotonicity**: $i_{\max}$ and $i_{\min}$ are forward-invariant

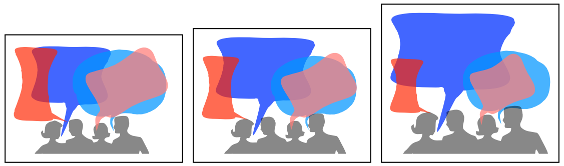
$$i_{\max} = \operatorname{argmax}_j \frac{x_j(0)}{x_j^*}$$

$$\implies \qquad i_{\max} = \operatorname{argmax}_j \frac{x_j(s)}{x_j^*}, \quad \text{for all subsequent } s$$

3. convergence via variation on classic **"max-min" Lyapunov function**:

$$V(x) = \max_j \left(\ln \frac{x_j}{x_j^*} \right) - \min_j \left(\ln \frac{x_j}{x_j^*} \right) \qquad \text{strictly decreasing for } x \neq x^*$$

Summary (Social Influence)



New perspective on influence networks and social power

- dynamics and feedback in influence networks
- novel mechanism for power accumulation / emergence of autocracy
- *measurement models and empirical validation*

Open directions

- robustness for distinct models of opinion dynamics and appraisal
- cognitive models for time-varying interpersonal appraisals
- appraisals and power accumulation mechanisms

Incomplete references on social power

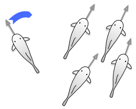
Social Influence

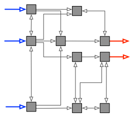
- J. R. P. French. A formal theory of social power,*Psychological Review*, 63 (1956), pp. 181–194.
- R. P. Abelson. Mathematical models of the distribution of attitudes under controversy, *Contributions to Mathematical Psychology*, 1964, pp. 142–160.
- V. Gecas and M. L. Schwalbe. Beyond the looking-glass self: Social structure and efficacy-based self-esteem. *Social Psychology Quarterly*, 46 (1983), pp. 77–88.
- N. E. Friedkin. A formal theory of reflected appraisals in the evolution of power. *Administrative Science Quarterly*, 56 (2011), pp. 501–529.

Our recent work

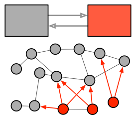
- P. Jia, A. MirTabatabaei, N. E. Friedkin, and F. Bullo. Opinion Dynamics and The Evolution of Social Power in Influence Networks. *SIAM Review*, 57(3):367-397, 2015.
- P. Jia, N. E. Friedkin, and F. Bullo. The Coevolution of Appraisal and Influence Networks leads to Structural Balance. *IEEE Transactions on Network Science and Engineering*, 3(4):286-298, 2016
- A. MirTabatabaei and F. Bullo. Opinion Dynamics in Heterogeneous Networks: Convergence Conjectures and Theorems. *SIAM Journal on Control and Optimization*, 50(5):2763-2785, 2012.

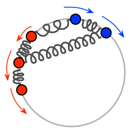
Network systems in science and technology

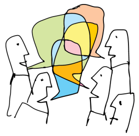

averaging


compartmental flows


mutualism


virus spread


coupled oscillators


social power

- **Models, behaviors, tools, and applications**
  PF and algebraic graphs for linear behaviors
  variety of nonlinearities — elegant methods and broad impact
- **Power Networks** and **Social Influence**
  fundamental prototypical problems
  nonlinear variations from linear framework
  key outstanding questions remain
- **Outreach and collaboration opportunity for control community**
  biologists, ecologists, economists, physicists ...