On the Dynamics of Opinions and Influence Systems

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Workshop on Distributed Control and Multi-Agent Systems

The Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science (AMSS), Chinese Academy of Sciences

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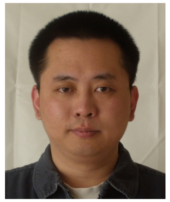
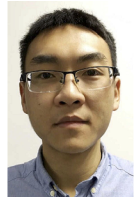
Acknowledgments



Peng Jia
Discover Financial

Ana MirTabatabaei
Apple

Wenjun Mei
ETH



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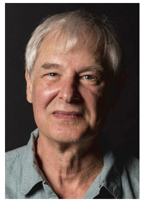

Noah E. Friekin
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ISS, AMSS, CAS

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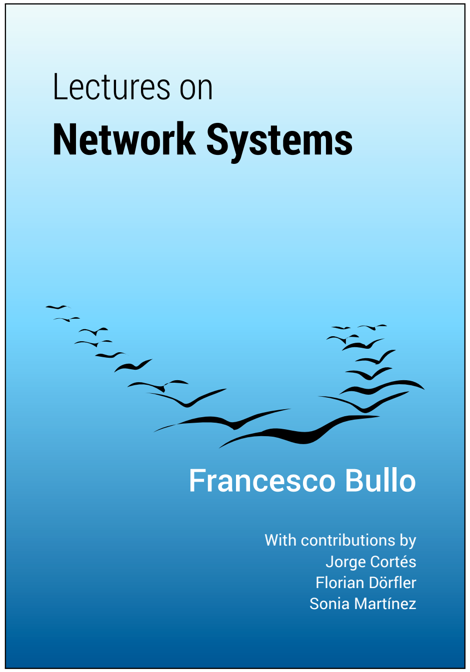
New text "Lectures on Network Systems"



Lectures on Network Systems, Francesco Bullo, Createspace, 1 edition, ISBN 978-1-986425-64-3

For students: free PDF for download
For instructors: slides and answer keys
http://motion.me.ucsb.edu/book-lns
https://www.amazon.com/dp/1986425649
300 pages (plus 200 pages solution manual)
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Linear Systems:

1. social, sensor, robotic & compartmental examples,
2. matrix and graph theory, with an emphasis on Perron–Frobenius theory and algebraic graph theory,
3. averaging algorithms in discrete and continuous time, described by static and time-varying matrices, and
4. positive & compartmental systems, dynamical flow systems, Metzler matrices.

Nonlinear Systems:

5. nonlinear consensus models,
6. population dynamic models in multi-species systems,
7. coupled oscillators, with an emphasis on the Kuramoto model and models of power networks

Educational introduction to network systems

What are fundamental dynamic phenomena over networks?

Examples drawn from:

- social networks
- Markov chains
- epidemic propagation
- population dynamic models
- evolutionary game theory
- parallel computing
- dynamical flow systems: transmission and traffic networks
- coupled oscillators
- multi-agent coordination
- network science

Dynamics and learning in social systems

Dynamic phenomena on dynamic social networks

1. dynamics: opinion formation,
   but also information propagation, task execution, strategic network formation
2. interpersonal network structures: influence systems,
   but also appraisal systems, transactive memory systems and other group psychological constructs

Questions on collective intelligence and rationality:

- wisdom of crowds vs. group think
- influence centrality (democracy versus autocracy)

Selected literature on math sociology and systems/control

M. O. Jackson. *Social and Economic Networks.*
Princeton University Press, 2010.
ISBN 0691148201

D. Easley and J. Kleinberg. *Networks, Crowds, and Markets: Reasoning About a Highly Connected World.*
Cambridge University Press, 2010.
ISBN 0521195330

exploding literature on social networks from sociology, physics, CS/engineering

Selected literature on opinion dynamics

J. R. P. French. A formal theory of social power.
Psychological Review, 63(3):181–194, 1956.
doi:10.1037/h0046123

M. H. DeGroot. Reaching a consensus.
Journal of the American Statistical Association, 69(345):118–121, 1974.
doi:10.1080/01621459.1974.10480137

N. E. Friedkin and E. C. Johnsen. Social influence and opinions.
Journal of Mathematical Sociology, 15(3-4):193–206, 1990.
doi:10.1080/0022250X.1990.9990069

F. Harary. A criterion for unanimity in French's theory of social power.
In D. Cartwright, editor, *Studies in Social Power*, pages 168–182. University of Michigan, 1959.
ISBN 0879442301.
URL http://psycnet.apa.org/psycinfo/1960-06701-006

Characterization of average consensus, 15 years before DeGroot

> *Theorem 14.* A strong group attains unanimity at the arithmetic mean of the initial opinions if and only if its matrix M is doubly stochastic.

A. V. Proskurnikov and R. Tempo. A tutorial on modeling and analysis of dynamic social networks. Part I.
Annual Reviews in Control, 43:65–79, 2017.
doi:10.1016/j.arcontrol.2017.03.002

Outline

1. Influence systems: basic models and statistical results on empirical data

   N. E. Friedkin, P. Jia, and F. Bullo. *A theory of the evolution of social power: Natural trajectories of interpersonal influence systems along issue sequences.* *Sociological Science, 3:444–472, 2016.* *doi:10.15195/v3.a20*

   N. E. Friedkin and F. Bullo. *How truth wins in opinion dynamics along issue sequences.* *Proceedings of the National Academy of Sciences, 114(43): 11380–11385, 2017.* *doi:10.1073/pnas.1710603114*

2. Influence systems: the mathematics of social power

Opinion dynamics and social power along sequences

Deliberative groups in social organization

- government: juries, panels, committees
- corporations: board of directors
- universities: faculty meetings

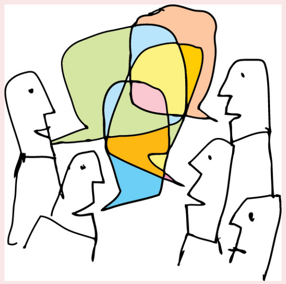
Natural social processes along sequences

- opinion dynamics for single issue?
- levels of openness and closure along sequence?
- influence accorded to others? emergence of leaders?

Groupthink = "deterioration of mental efficiency ... from in-group pressures," by I. Janis, 1972

Wisdom of crowds = "group aggregation of information results in better decisions than individual's" by J. Surowiecki, 2005

Postulated mechanisms for opinion dynamics 1/2



French-DeGroot averaging model

$$y_i^+ := \text{average}\big(y_i, \{y_j, j \text{ is neighbor of } i\}\big)$$

$$y(k+1) = Ay(k)$$

where A is nonnegative and row-stochastic
Consensus under mild connectivity assumptions:

$$\lim_{k\to\infty} y(k) = (c^\top y(0))\, \mathbb{1}_n$$

self-weight = level of closure:	a_{ii}	diagonal entries of influence matrix
social power:	c_i	entries of dominant left eigenvector

Postulated mechanisms for opinion dynamics 2/2

Averaging (French-DeGroot model)

$$y(k+1) = Ay(k) \qquad \lim_{k\to\infty} y(k) = (c^\top y(0))\mathbb{1}_n$$

Averaging + attachment to initial opinion (F-J model)

$$y(k+1) = (I_n - \Lambda)Ay(k) + \Lambda y(0), \quad \Lambda = \text{diag}(A)$$

Convergence under mild connectivity+stubburness assumptions:

$$\lim_{k\to\infty} y(k) = V \cdot y(0), \qquad \text{for } V = (I_n - (I_n - \Lambda)A)^{-1}\Lambda$$

$$c = V^\top \mathbb{1}_n/n = \text{average contribution of each agent}$$

self-weight = level of closure:	a_{ii}	diagonal entries of influence matrix
social power:	c_i	entries of centrality vector

Today we skip these proofs

Analysis of French-DeGroot and F-J models well-understood:

- Jordan normal form
- Perron-Frobenius theory
- algebraic graph theory (connectivity, periodicity, etc)

Experiments on opinion formation and influence networks

domains: risk/reward choice, analytical reliability, resource allocation

- **30 groups of 4 subjects** in a face-to-face discussion
- **sequence of 15 issues**
- each issue is **risk/reward choice**:

  what is your minimum level of confidence (scored 0-100) required to accept a risky option with a high payoff rather than a less risky option with a low payoff?
  e.g.: medical, financial, professional, etc

- **"please, reach consensus"** pressure

- On each issue, each subject recorded (privately/chronologically):
  1. **an initial opinion** prior to the-group discussion,
  2. **a final opinion** after the group-discussion (3-27 mins),
  3. **an allocation of "100 influence units"**
     ("these allocations represent your appraisal of the relative influence of each group member's opinion on yours").

(1/3) Prediction of individual final opinions

Balanced random-intercept multilevel longitudinal regression

	(a)	(b)	(c)
F-J prediction		0.897***	1.157***
		(0.018)	(0.032)
initial opinions			−0.282***
			(0.031)
log likelihood	-8579.835	-7329.003	-7241.097

Standard errors are in parentheses; ** $p \leq 0.01$, *** $p \leq 0.001$; maximum likelihood estimation with robust standard errors; $n = 1,800$.

FJ averaging model is predictive for risk/reward choice issues

Extensions to: intellective and resource allocation issues

Risk/reward choice

Intellective issue = Problem solving

Two medical teams are working independently to achieve a cure for a disease.
Team A succeeds if
problems A_1 and A_2 with $\mathbb{P}[A_1] = 0.60$ and $\mathbb{P}[A_2] = 0.45$.
Team B succeeds if
problems B_1, B_2, and B_3, with $\mathbb{P}[B_1] = 0.80$, $\mathbb{P}[B_2] = 0.85$, $\mathbb{P}[B_3] = 0.95$
What is your estimate of the probability that the disease will be cured?

Multidimensional resource allocation

Diet problem: Given 4 food groups: Fruits, Vegetables, Grains, and Meats.
What do you recommend as min and max percent of food consumption in terms of (1) Fruits or Vegetables, (2) Grains, and (3) Meats?
What are your ideal percentages in your preferred min/max ranges?

Opinion averaging models are predictive

Sociological Sciences 2016

N. E. Friedkin and F. Bullo. How truth wins in opinion dynamics along issue sequences.
Proceedings of the National Academy of Sciences, 114(43):11380–11385, 2017.
doi:10.1073/pnas.1710603114

Empirical evidence that (1) FJ model substantially clarifies how truth wins in groups engaged in sequences of intellective issues (2) learning and reflected appraisal take place

N. E. Friedkin, W. Mei, A. V. Proskurnikov, and F. Bullo. Mathematical structures in group decision-making on resource allocation distributions.
Submitted, November 2017.
Submitted

Empirical evidence that (1) FJ model provides quantitative mechanistic explanation for uncertain multi-objective decision making problem and (2) FJ provides detailed explanation for group satisficing solutions

Opinion dynamics along sequences
Postulated mechanism for network evolution

From Wikipedia

1. *Reflected appraisal = a person's perception of how others see and evaluate him or her.*
2. *This process has been deemed important to the development of a person's self-esteem, because it includes interaction with people outside oneself.*
3. *The reflected appraisal process concludes that people come to think of themselves in the way they believe others think of them.*

Reflected appraisal process (Cooley 1902 and Friedkin 2011)

Along issues $s = 1, 2, \dots$, individual dampens/elevates self-weight according to prior influence centrality

self-weights $:=$ *relative control on prior issues* $=$ *social power*

(2/3) Prediction of individual level of closure

Balanced random-intercept multilevel longitudinal regression

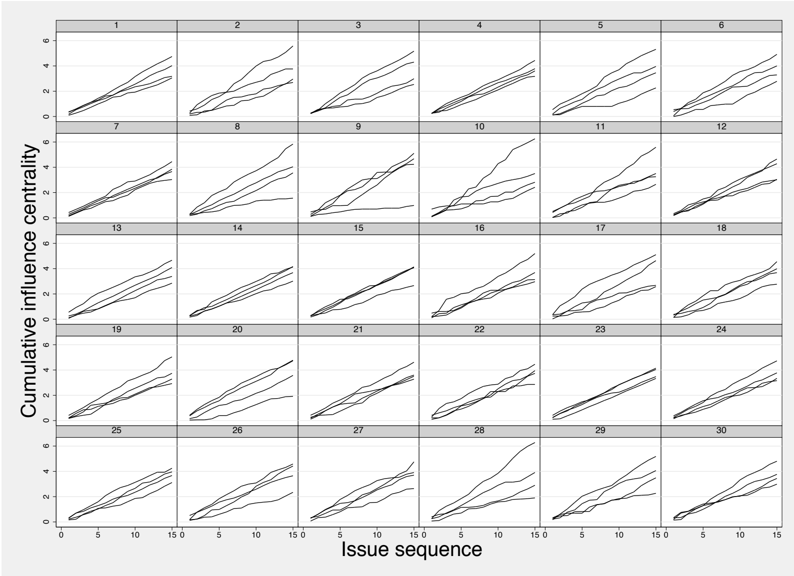
individual's "closure to influence" as predicted by:

- individual's prior centrality $c_i(s)$
- individual's time-averaged centrality $\bar{c}_i(s) = \frac{1}{s}\sum_{t=1}^{s} c_i(t)$

	(a)	(b)	(c)
$c_i(s)$		0.336***	
$\bar{c}_i(s)$			0.404**
s		0.002	−0.018***
$s \times c_i(s)$		0.171	
$s \times \bar{c}_i(s)$			0.095***
log likelihood	-367.331	-327.051	-293.656

prior and cumulative prior centrality predicts individual closure

(3/3) Prediction of cumulative influence centrality



individuals accumulate influence centralities at different rates, and their time-average centrality stabilizes to constant values

Outline

1. Influence systems: statistical results on empirical data

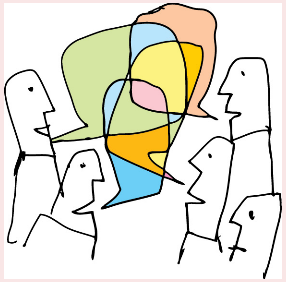
2. Influence systems: the mathematics of social power

P. Jia, A. MirTabatabaei, N. E. Friedkin, and F. Bullo. Opinion dynamics and the evolution of social power in influence networks. SIAM Review, 57(3):367–397, 2015.
doi:10.1137/130913250

P. Jia, N. E. Friedkin, and F. Bullo. Opinion dynamics and social power evolution over reducible influence networks. SIAM Journal on Control and Optimization, 55(2):1280–1301, 2017.
doi:10.1137/16M1065677

G. Chen, X. Duan, N. E. Friedkin, and F. Bullo. Social power dynamics over switching and stochastic influence networks. IEEE Transactions on Automatic Control, May 2017.
doi:10.1109/TAC.2018.2822182.
To appear

Opinion dynamics and social power along issue sequences



French-DeGroot averaging model

$$y(k+1) = Ay(k)$$

Consensus under mild assumptions:

$$\lim_{k\to\infty} y(k) = (v_{\text{left}}(A)\cdot y(0))\mathbb{1}_n$$

where $v_{\text{left}}(A)$ is **social power**

- $A_{ii} =: x_i$ are **self-weights / self-appraisal = level of closure**
- let W_{ij} be **relative interpersonal accorded weights**
  define $A_{ij} =: (1-x_i)W_{ij}$ so that

$$A(x) = \text{diag}(x) + \text{diag}(\mathbb{1}_n - x)W$$

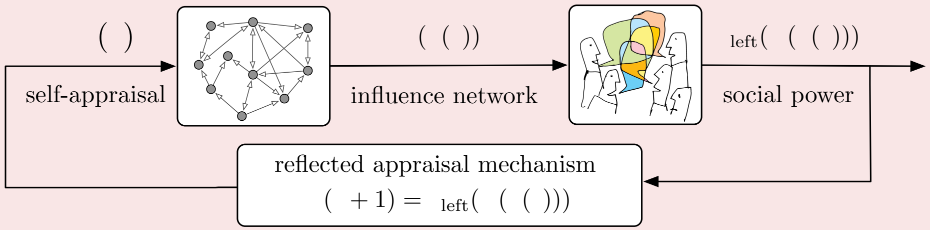
- $v_{\text{left}}(W) = (w_1, \dots, w_n)$ = dominant eigenvector for W

Opinion dynamics and social power along issue sequences

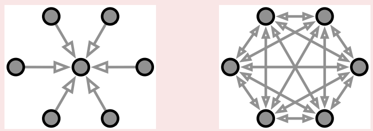
Reflected appraisal phenomenon (Cooley 1902 and Friedkin 2011)

along issues $s = 1, 2, \dots,$ individual dampens/elevates self-weight according to prior influence centrality

self-weights ⇐ relative control on prior issues = social power



Dynamics of the influence network



Existence and stability of equilibria?
Role of network structure and parameters?
Emergence of *autocracy* and *democracy*?

Theorem: For strongly connected W and non-trivial initial conditions

1. **unique fixed point** $x^* = x^*(w_1, \dots, w_n)$
2. **convergence = forgets initial condition**

$$\lim_{s\to\infty} x(s) = \lim_{s\to\infty} v_{\text{left}}(A(x(s))) = x^*$$

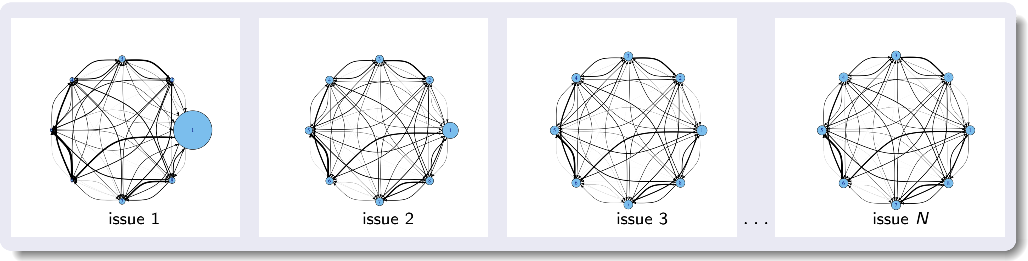
3. **accumulation of social power and self-appraisal**
   - fixed point x^* has same ordering of $(w_1, \dots, w_n)$
   - x^* is an extreme version of $(w_1, \dots, w_n)$

Emergence of democracy

If W is doubly-stochastic:

1. the non-trivial fixed point is $\frac{\mathbb{1}_n}{n}$
2. $\lim_{s\to\infty} x(s) = \lim_{s\to\infty} v_{\text{left}}(A(x(s))) = \frac{\mathbb{1}_n}{n}$

- Uniform social power
- No power accumulation = evolution to democracy

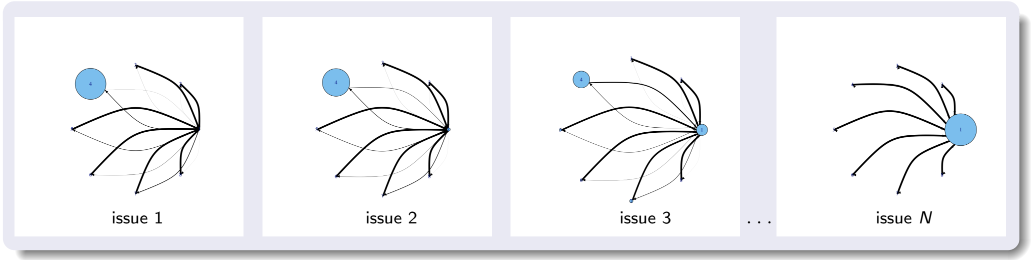


Emergence of autocracy

If W has star topology with center j:

1. there are no non-trivial fixed points
2. $\lim_{s\to\infty} x(s) = \lim_{s\to\infty} v_{\text{left}}(A(x(s))) = \mathbb{e}_j$

- Autocrat appears in center node of star topology
- Extreme power accumulation = evolution to autocracy

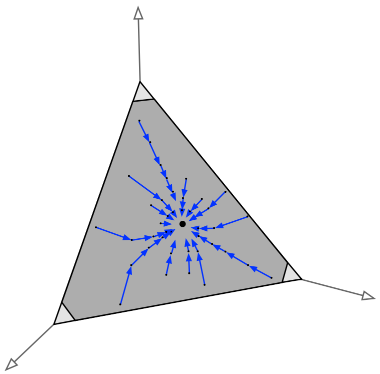


Analysis methods

1. existence of x^* via
   Brower fixed point theorem
2. **monotonicity**:
   $i_{\max}$ and $i_{\min}$ are forward-invariant

$$i_{\max} = \operatorname{argmax}_j \frac{x_j(0)}{x_j^*}$$

$$\implies \quad i_{\max} = \operatorname{argmax}_j \frac{x_j(s)}{x_j^*}, \quad \text{for all subsequent } s$$



3. convergence via variation on classic **"max-min" Lyapunov function**:

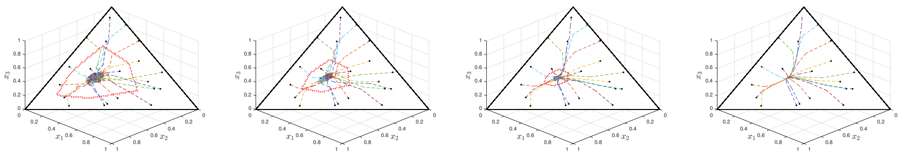
$$V(x) = \max_j \left(\ln \frac{x_j}{x_j^*}\right) - \min_j \left(\ln \frac{x_j}{x_j^*}\right) \qquad \text{strictly decreasing for } x \neq x^*$$

Reducible interpersonal networks

- W reducible
- two cases: single sink and multiple sinks in condensation
- generalized analysis with similar and related results

Stochastic models with cumulative memory

1. assume noisy interpersonal weights $W(s) = W_0 + N(s)$
   assume noisy perception of social power
   $x(s+1) = v_{\text{left}}(A(x(s))) + n(s)$
   Thm: practical stability of x^*



2. assume self-weight := cumulative average of prior social power

$$x(s+1) = (1-\alpha(s))x(s) + \alpha(s)\Big(v_{\text{left}}(A(x(s))) + n(s)\Big)$$

   Thm: a.s. convergence to x^* (under technical conditions)

Recent extensions on social power evolution

- X. Chen, J. Liu, M.-A. Belabbas, Z. Xu, and T. Başar. Distributed evaluation and convergence of self-appraisals in social networks.
  IEEE Transactions on Automatic Control, 62(1):291–304, 2017.
  doi:10.1109/TAC.2016.2554280
- M. Ye, J. Liu, B. D. O. Anderson, C. Yu, and T. Başar. Evolution of social power in social networks with dynamic topology.
  IEEE Transactions on Automatic Control, 2018.
  doi:10.1109/TAC.2018.2805261.
  To appear
- Z. Askarzadeh, R. Fu, A. Halder, Y. Chen, and T. T. Georgiou. Stability theory in ℓ_1 for nonlinear Markov chains and stochastic models for opinion dynamics, June 2017.
  URL https://arxiv.org/pdf/1706.03158

Summary

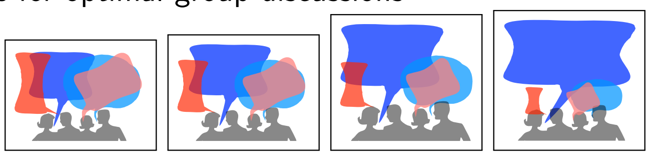
New perspective on influence networks and social power

- designed/executed/analyzed experiments on group discussions
- proposed/analyzed/validated dynamical models with feedback
- novel mechanism for power accumulation / emergence of autocracy

Open directions

- robustness to modelling assumptions
- dynamics of interpersonal appraisals
- larger-scale online experiments

- intervention strategies for optimal group discussions



No one speaks twice, until everyone speaks once
Robert's Rules of Order & parliamentary procedures