On the Dynamics of Influence and Appraisal Networks

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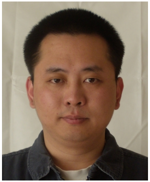
Acknowledgments



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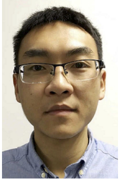

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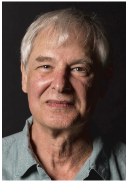

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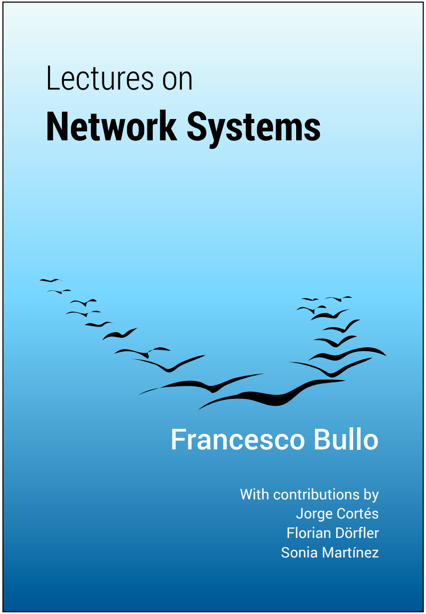
New text “Lectures on Network Systems”



Lectures on Network Systems, 1 edition
ISBN 978-1-986425-64-3

For students: free PDF for download
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http://motion.me.ucsb.edu/book-lns
https://www.amazon.com/dp/1986425649
300 pages (plus 200 pages solution manual)
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Linear Systems:

1. social, sensor, robotic & compartmental examples,
2. matrix and graph theory, with an emphasis on Perron–Frobenius theory and algebraic graph theory,
3. averaging algorithms in discrete and continuous time, described by static and time-varying matrices, and
4. positive & compartmental systems, dynamical flow systems, Metzler matrices.

Nonlinear Systems:

5. nonlinear consensus models,
6. population dynamic models in multi-species systems,
7. coupled oscillators, with an emphasis on the Kuramoto model and models of power networks

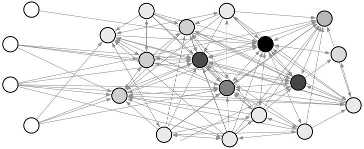
Dynamics and learning in social systems

Dynamic phenomena on dynamic social networks

1. opinion formation, information propagation, collective learning, task decomposition/allocation/execution
2. interpersonal network structures, e.g., influences & appraisals

Questions on collective intelligence, rationality & performance:

- wisdom of crowds vs. group think
- influence centrality (democracy versus autocracy)
- collective learning or lack thereof



Dynamics and learning in social systems
Mathematical sociology + systems/controls

opinion dynamics over influence networks

- seminal works: French '56, Harary '59, DeGroot '74, Friedkin '90
- recently: bounded confidence, learning, social power
- key object: row stochastic matrix

dynamics of appraisal networks and balance theory

- seminal works: Heider '46, Cartwright '56, Davis/Leinhardt '72
- recently: dynamic balance, empirical studies
- key object: signed matrix

Not considered today:

- other dynamic phenomena (epidemics)
- static network science (clustering)
- game theory and strategic behavior (network formation)

Selected literature on math sociology and systems/control

F. Harary, R. Z. Norman, and D. Cartwright. *Structural Models: An Introduction to the Theory of Directed Graphs.*
John Wiley & Sons, 1965.
ISBN 047135130X (Institute for Social Research, University of Michigan)

M. O. Jackson. *Social and Economic Networks.*
Princeton University Press, 2010.
ISBN 0691148201

D. Easley and J. Kleinberg. *Networks, Crowds, and Markets: Reasoning About a Highly Connected World.*
Cambridge University Press, 2010.
ISBN 0521195330

N. E. Friedkin and E. C. Johnsen. *Social Influence Network Theory: A Sociological Examination of Small Group Dynamics.*
Cambridge University Press, 2011.
ISBN 9781107002463

exploding literature on social networks from sociology, physics, CS/engineering

Selected literature on opinion dynamics

J. R. P. French. A formal theory of social power.
Psychological Review, 63(3):181–194, 1956.
doi:10.1037/h0046123

M. H. DeGroot. Reaching a consensus.
Journal of the American Statistical Association, 69(345):118–121, 1974.
doi:10.1080/01621459.1974.10480137

N. E. Friedkin and E. C. Johnsen. Social influence and opinions.
Journal of Mathematical Sociology, 15(3-4):193–206, 1990.
doi:10.1080/0022250X.1990.9990069

A. V. Proskurnikov and R. Tempo. A tutorial on modeling and analysis of dynamic social networks. Part I.
Annual Reviews in Control, 43:65–79, 2017.
doi:10.1016/j.arcontrol.2017.03.002

Selected literature on social power & reflected appraisal

C. H. Cooley. *Human Nature and the Social Order.*
Charles Scribner Sons, New York, 1902

V. Gecas and M. L. Schwalbe. Beyond the looking-glass self: Social structure and efficacy-based self-esteem.
Social Psychology Quarterly, 46(2):77–88, 1983.
URL `http://www.jstor.org/stable/3033844`

N. E. Friedkin. A formal theory of reflected appraisals in the evolution of power.
Administrative Science Quarterly, 56(4):501–529, 2011.
doi:10.1177/0001839212441349

Outline

1. Influence systems: statistical results on empirical data

   N. E. Friedkin, P. Jia, and F. Bullo. A theory of the evolution of social power: Natural trajectories of interpersonal influence systems along issue sequences.
   Sociological Science, 3:444–472, 2016.
   doi:10.15195/v3.a20

   N. E. Friedkin and F. Bullo. How truth wins in opinion dynamics along issue sequences.
   Proceedings of the National Academy of Sciences, 114(43):11380–11385, 2017.
   doi:10.1073/pnas.1710603114

2. Influence systems: the mathematics of social power

3. Appraisal systems and collective learning

Opinion dynamics and social power along sequences

Deliberative groups in social organization

- government: juries, panels, committees
- corporations: board of directors
- universities: faculty meetings

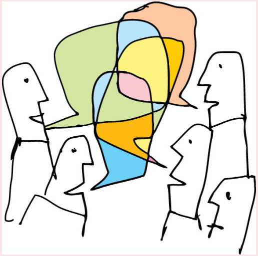
Natural social processes along sequences

- opinion dynamics for single issue?
- levels of openness and closure along sequence?
- influence accorded to others? emergence of leaders?

Groupthink = "deterioration of mental efficiency . . . from in-group pressures," by I. Janis, 1972

Wisdom of crowds = "group aggregation of information results in better decisions than individual's" by J. Surowiecki, 2005

Postulated mechanisms for opinion dynamics 1/2



French-DeGroot averaging model

$$y_i^+ := \text{average}\big(y_i, \{y_j, j \text{ is neighbor of } i\}\big)$$

$$\Downarrow$$

$$y(k+1) = Ay(k)$$

where A is nonnegative and row-stochastic
Consensus under mild connectivity assumptions:

$$\lim_{k\to\infty} y(k) = \left(c^\top y(0)\right) \mathbb{1}_n$$

self-weight = level of closure:	a_{ii}	diagonal entries of influence matrix
social power:	c_i	entries of dominant left eigenvector

Postulated mechanisms for opinion dynamics 2/2

Averaging (French-DeGroot model)

$$y(k+1) = Ay(k) \qquad \lim_{k\to\infty} y(k) = (c^\top y(0))\mathbb{1}_n$$

Averaging + attachment to initial opinion (F-J model)

$$y(k+1) = (I_n - \Lambda)Ay(k) + \Lambda y(0),$$
$$\Lambda = \mathrm{diag}(A)$$

Convergence under mild connectivity+stubburness assumptions:

$$\lim_{k\to\infty} y(k) = V \cdot y(0), \qquad \text{for } V = (I_n - (I_n - \Lambda)A)^{-1}\Lambda$$

$$c = V^\top \mathbb{1}_n / n = \text{average contribution of each agent}$$

self-weight = level of closure: a_{ii} diagonal entries of influence matrix

social power: c_i entries of centrality vector

Today we skip these proofs

Mathematical analysis of French-DeGroot and F-J models is well understood:

- Jordan normal form
- Perron-Frobenius theory
- algebraic graph theory (connectivity, periodicity, etc)

Experiments on opinion formation and influence networks

domains: risk/reward choice, analytical reliability, resource allocation

- **30 groups of 4 subjects** in a face-to-face discussion
- **sequence of 15 issues**
- each issue is **risk/reward choice**:

  what is your minimum level of confidence (scored 0-100) required to accept a risky option with a high payoff rather than a less risky option with a low payoff?
  e.g.: medical, financial, professional, etc

- **"please, reach consensus"** pressure

- On each issue, each subject recorded (privately/chronologically):
  1. **an initial opinion** prior to the-group discussion,
  2. **a final opinion** after the group-discussion (3-27 mins),
  3. **an allocation of "100 influence units"**
     ("these allocations represent your appraisal of the relative influence of each group member's opinion on yours").

(1/3) Prediction of individual final opinions

Balanced random-intercept multilevel longitudinal regression

	(a)	(b)	(c)
F-J prediction		0.897^{***}	1.157^{***}
		(0.018)	(0.032)
initial opinions			-0.282^{***}
			(0.031)
log likelihood	-8579.835	-7329.003	-7241.097

Standard errors are in parentheses; ** $p \leq 0.01$, *** $p \leq 0.001$; maximum likelihood estimation with robust standard errors; $n = 1,800$.

FJ averaging model is predictive for risk/reward choice issues

Extensions to: intellective and resource allocation issues

Risk/reward choice: *N. E. Friedkin, P. Jia, and F. Bullo. A theory of the evolution of social power: Natural trajectories of interpersonal influence systems along issue sequences.*

Sociological Science, 3:444–472, 2016.

doi:10.15195/v3.a20

Intellective issue

Two medical teams are working independently to achieve a cure for a disease.

Team A succeeds if

problems A_1 *and* A_2 *with* $\mathbb{P}[A_1] = 0.60$ *and* $\mathbb{P}[A_2] = 0.45$.

Team B succeeds if

problems B_1, B_2, *and* B_3, *with* $\mathbb{P}[B_1] = 0.80$, $\mathbb{P}[B_2] = 0.85$, $\mathbb{P}[B_3] = 0.95$

What is your estimate of the probability that the disease will be cured?

Multidimensional resource allocation

Diet problem: Given 4 food groups: Fruits, Vegetables, Grains, and Meats.

What do you recommend as min and max percent of food consumption in terms of (1) Fruits or Vegetables, (2) Grains, and (3) Meats?

What are your ideal percentages in your preferred min/max ranges?

Recent empirical and theoretical results

Averaging models are predictive

N. E. Friedkin, P. Jia, and F. Bullo. A theory of the evolution of social power: Natural trajectories of interpersonal influence systems along issue sequences.
Sociological Science, 3:444–472, 2016.
doi:10.15195/v3.a20

N. E. Friedkin and F. Bullo. How truth wins in opinion dynamics along issue sequences.
Proceedings of the National Academy of Sciences, 114(43):11380–11385, 2017.
doi:10.1073/pnas.1710603114

> *Empirical evidence that (1) FJ model substantially clarifies how truth wins in groups engaged in sequences of intellective issues (2) learning and reflected appraisal take place*

N. E. Friedkin, W. Mei, A. V. Proskurnikov, and F. Bullo. Mathematical structures in group decision-making on resource allocation distributions.
Submitted

> *Empirical evidence that (1) FJ model provides quantitative mechanistic explanation for uncertain multi-objective decision making problem and (2) FJ provides detailed explanation for group satisficing solutions*

From Wikipedia

1. Reflected appraisal = a person's perception of how others see and evaluate him or her.

2. This process has been deemed important to the development of a person's self-esteem, because it includes interaction with people outside oneself.

3. The reflected appraisal process concludes that people come to think of themselves in the way they believe others think of them.

Reflected appraisal process (Cooley 1902 and Friedkin 2011)

Along issues $s = 1, 2, \ldots,$ individual dampens/elevates self-weight according to prior influence centrality

self-weights $:=$ *relative control on prior issues* $=$ *social power*

(2/3) Prediction of individual level of closure

Balanced random-intercept multilevel longitudinal regression

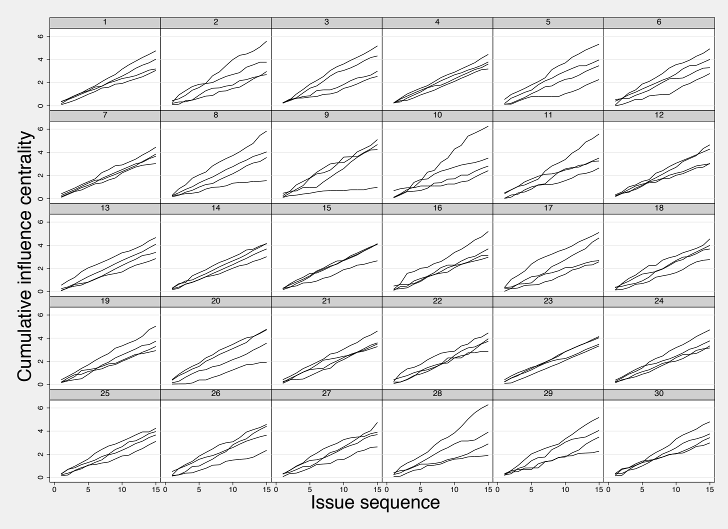
individual's "closure to influence" as predicted by:

- individual's prior centrality $c_i(s)$
- individual's time-averaged centrality $\bar{c}_i(s) = \frac{1}{s}\sum_{t=1}^{s} c_i(t)$

	(a)	(b)	(c)
$c_i(s)$		0.336^{***}	
$\bar{c}_i(s)$			0.404^{**}
s		0.002	-0.018^{***}
$s \times c_i(s)$		0.171	
$s \times \bar{c}_i(s)$			0.095^{***}
log likelihood	-367.331	-327.051	-293.656

prior and cumulative prior centrality predicts individual closure

(3/3) Prediction of cumulative influence centrality



individuals accumulate influence centralities at different rates, and their time-average centrality stabilizes to constant values

Outline

1. Influence systems: statistical results on empirical data

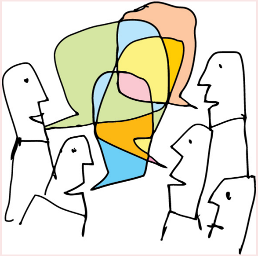
2. **Influence systems: the mathematics of social power**

   P. Jia, A. MirTabatabaei, N. E. Friedkin, and F. Bullo. Opinion dynamics and the evolution of social power in influence networks. SIAM Review, 57(3):367–397, 2015. doi:10.1137/130913250

   G. Chen, X. Duan, N. E. Friedkin, and F. Bullo. Social power dynamics over switching and stochastic influence networks. IEEE Transactions on Automatic Control, May 2017. doi:10.1109/TAC.2018.2822182

3. Appraisal systems and collective learning

Opinion dynamics and social power along issue sequences



French-DeGroot averaging model

$$y(k+1) = Ay(k)$$

Consensus under mild assumptions:

$$\lim_{k\to\infty} y(k) = (v_{\text{left}}(A)\cdot y(0))\mathbb{1}_n$$

where $v_{\text{left}}(A)$ is **social power**

- $A_{ii} =: x_i$ are **self-weights / self-appraisal = level of closure**
- let W_{ij} be **relative interpersonal accorded weights**
  define $A_{ij} =: (1-x_i)W_{ij}$ so that

$$A(x) = \operatorname{diag}(x) + \operatorname{diag}(\mathbb{1}_n - x)W$$

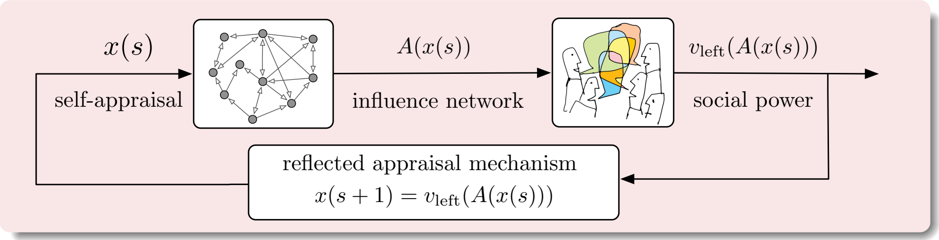
- $v_{\text{left}}(W) = (w_1, \ldots, w_n) =$ dominant eigenvector for W

Opinion dynamics and social power along issue sequences

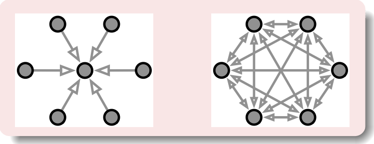
Reflected appraisal phenomenon (Cooley 1902 and Friedkin 2011)

> *along issues $s = 1, 2, \dots,$ individual dampens/elevates self-weight according to prior influence centrality*

self-weights ⇐ relative control on prior issues = social power



Dynamics of the influence network



Existence and stability of equilibria?
Role of network structure and parameters?
Emergence of *autocracy* and *democracy*?

Theorem: For strongly connected W and non-trivial initial conditions

1. **unique fixed point** $x^* = x^*(w_1, \ldots, w_n)$
2. **convergence = forgets initial condition**

$$\lim_{s\to\infty} x(s) = \lim_{s\to\infty} v_{\text{left}}(A(x(s))) = x^*$$

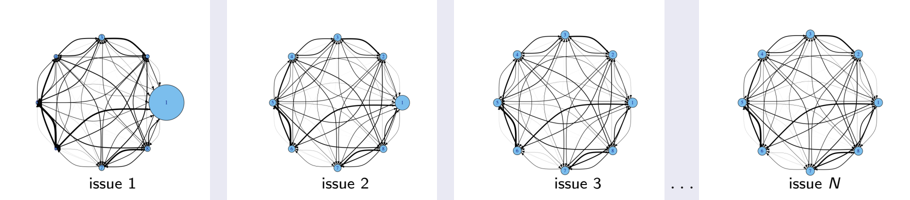
3. **accumulation of social power and self-appraisal**
   - fixed point x^* has same ordering of $(w_1, \ldots, w_n)$
   - x^* is an extreme version of $(w_1, \ldots, w_n)$

Emergence of democracy

If W is doubly-stochastic:

1. the non-trivial fixed point is $\dfrac{\mathbb{1}_n}{n}$
2. $\lim_{s\to\infty} x(s) = \lim_{s\to\infty} v_{\text{left}}(A(x(s))) = \dfrac{\mathbb{1}_n}{n}$

- Uniform social power
- No power accumulation = evolution to democracy

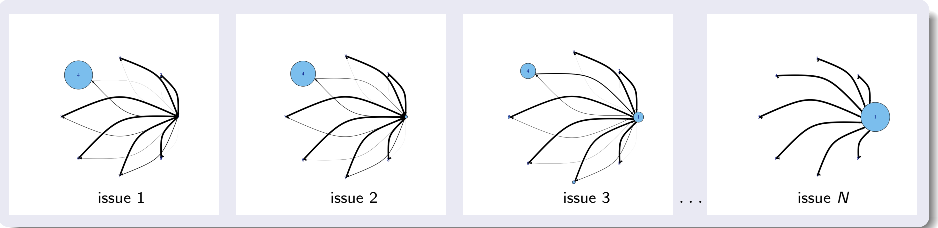


Emergence of autocracy

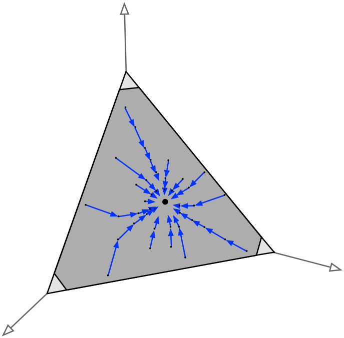
If W has star topology with center j:

1. there are no non-trivial fixed points
2. $\lim_{s\to\infty} x(s) = \lim_{s\to\infty} v_{\text{left}}(A(x(s))) = \mathbb{e}_j$

- Autocrat appears in center node of star topology
- Extreme power accumulation = evolution to autocracy



Analysis methods



1. existence of x^* via
   Brower fixed point theorem

2. **monotonicity**:
   $i_{\max}$ and $i_{\min}$ are forward-invariant

$$i_{\max} = \operatorname{argmax}_j \frac{x_j(0)}{x_j^*}$$

$$\implies \qquad i_{\max} = \operatorname{argmax}_j \frac{x_j(s)}{x_j^*}, \quad \text{for all subsequent } s$$

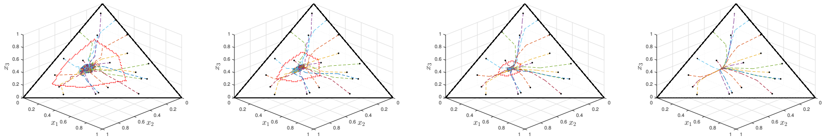
3. convergence via variation on classic **“max-min” Lyapunov function**:

$$V(x) = \max_j \left(\ln \frac{x_j}{x_j^*} \right) - \min_j \left(\ln \frac{x_j}{x_j^*} \right) \qquad \text{strictly decreasing for } x \neq x^*$$

Stochastic models with cumulative memory

Other extensions: modified models, reducible W, periodic W ...

1. assume noisy interpersonal weights $W(s) = W_0 + N(s)$
   assume noisy perception of social power
   $x(s+1) = v_{\text{left}}(A(x(s))) + n(s)$
   Thm: practical stability of x^*



2. assume self-weight := cumulative average of prior social power

$$x(s+1) = (1-\alpha(s))x(s) + \alpha(s)\Big(v_{\text{left}}(A(x(s))) + n(s)\Big)$$

Thm: a.s. convergence to x^* (under technical conditions)

Recent extensions on social power evolution

X. Chen, J. Liu, M.-A. Belabbas, Z. Xu, and T. Başar. Distributed evaluation and convergence of self-appraisals in social networks.
IEEE Transactions on Automatic Control, 62(1):291–304, 2017.
doi:10.1109/TAC.2016.2554280

M. Ye, J. Liu, B. D. O. Anderson, C. Yu, and T. Başar. On the analysis of the DeGroot-Friedkin model with dynamic relative interaction matrices.
In *IFAC World Congress*, pages 11902–11907, Toulouse, France, July 2017.
doi:10.1016/j.ifacol.2017.08.1426

P. Jia, N. E. Friedkin, and F. Bullo. Opinion dynamics and social power evolution over reducible influence networks.
SIAM Journal on Control and Optimization, 55(2):1280–1301, 2017.
doi:10.1137/16M1065677

Z. Askarzadeh, R. Fu, A. Halder, Y. Chen, and T. T. Georgiou. Stability theory in ℓ_1 for nonlinear Markov chains and stochastic models for opinion dynamics, 2017.
URL `https://arxiv.org/pdf/1706.03158`

Summary (Social Influence)

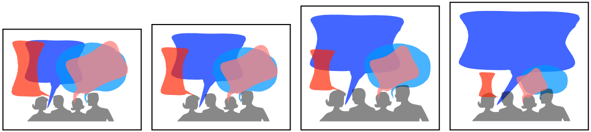
New perspective on influence networks and social power

- designed/executed/analyzed experiments on group discussions
- proposed/analyzed/validated dynamical models with feedback
- novel mechanism for power accumulation / emergence of autocracy

Open directions

- model robustness
- dynamics of interpersonal appraisals
- larger-scale online experiments

- intervention strategies for optimal group discussions



No one speaks twice, until everyone speaks once
Robert's Rules of Order & parliamentary procedures

Outline

1. Influence systems: statistical results on empirical data
2. Influence systems: the mathematics of social power
3. **Appraisal systems and collective learning**

   W. Mei, N. E. Friedkin, K. Lewis, and F. Bullo. Dynamic models of appraisal networks explaining collective learning.
   IEEE Transactions on Automatic Control, 2018.
   doi:10.1109/TAC.2017.2775963

Appraisal systems and collective learning

Teams and tasks

- individuals with skills
- executing a sequence of tasks
- related through networks of interpersonal appraisals and influence

Natural social processes along sequences

- how is task decomposed, assigned and executed?
- how do individuals learn about each other?
- how does group performance evolve?

models/conditions for learning correct appraisals and
achieving optimal assignments

model/conditions for failure to learn and correctly assign

Selected literature on learning in appraisal systems

D. M. Wegner. Transactive memory: A contemporary analysis of the group mind.
In B. Mullen and G. R. Goethals, editors, *Theories of Group Behavior*, pages 185–208. Springer, 1987.
doi:10.1007/978-1-4612-4634-3

K. Lewis. Measuring transactive memory systems in the field: Scale development and validation.
Journal of Applied Psychology, 88(4):587–604, 2003.
doi:10.1037/0021-9010.88.4.587

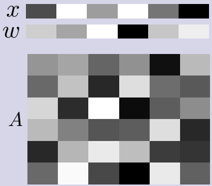
J. R. Austin. Transactive memory in organizational groups: the effects of content, consensus, specialization, and accuracy on group performance.
Journal of Applied Psychology, 88(5):866, 2003.
doi:10.1037/0021-9010.88.5.866

Collective Learning Model

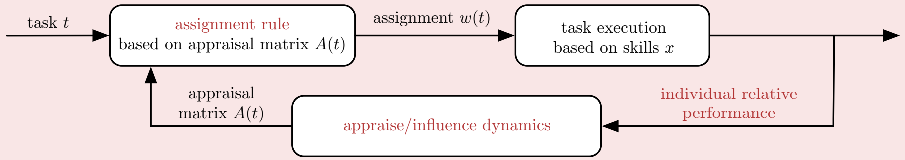
Transactive memory system (TMS)

- collective knowledge on who knows what
- task execution & observation lead group to increasingly accurate knowldge & consensus

1. n individuals with skill levels $x \in \Delta_n$
2. TMS = appraisal matrix row stochastic
3. workload assignment $w \in \Delta_n$
4. optimal assignment: $w^* = x$



Collective Learning Model: Key Assumptions



Key assumptions of assign/appraise dynamics

1. Static assignment by appraisal centrality or appraisal average
2. Static individual performance $p_i = x_i/w_i$
3. Observation of own vs average performance

   ϕ_i = own performance − average of observed subgroup performance
4. appraise/influence: elevate/dampen self-appraisal + opinion exchange

Collective Learning Model: Result 1/3

1. TMS is akin to a manager

- Along assign/appraise dynamics: generalized replicator equation

$$\dot{w}_i = w_i\left(a_{ii}\phi_i - \sum_k a_{kk} w_k \phi_k\right)$$

- akin to replicator equation modeling a "manager dynamics"

$$\dot{w}_i = w_i\left(p_i - \sum_k w_k p_k\right) \quad \Longrightarrow \quad w(t) \to x$$

Collective Learning Model: Result 2/3

1. TMS is akin to a manager
2. Opinion exchange compensates for lack of observation.

Conditions for asymptotic optimization

- assign/appraise dynamics:

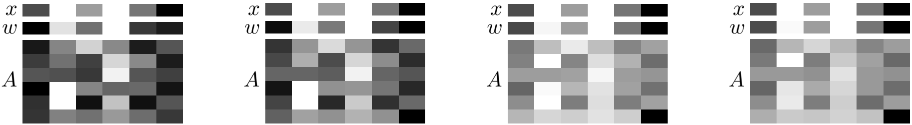
  strongly connected observation network $\quad \Rightarrow \quad w(t) \to w^* = x$

- additionally with influence dynamics:

  globally reachable node in observation network $\quad \Rightarrow \quad w(t) \to w^* = x$

  moreover: consensus on correct appraisals

assign/appraise dynamics:



assign/appraise + influence dynamics:

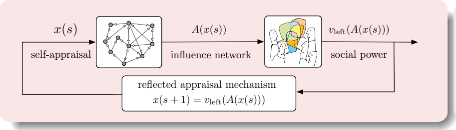
Collective Learning Model: Result 3/3

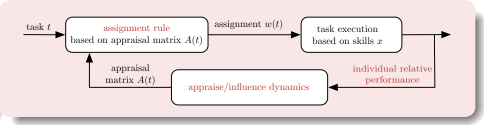
1. TMS is akin to a manager
2. Opinion exchange compensates for lack of observation.
3. Causes of incorrect learning and suboptimal assignment:
   - assignment rule: appraisal average (and no influence dynamics)
   - appraise dynamics: observation graph without connectivity properties
   - influence dynamics: prejudice model (F-J model)

Summary

Contributions

- dynamics and feedback in sociology and organization science
- a new perspective on social power, self-appraisal, influence networks
- a new explanation of team learning and rationality





Next steps

1. theoretical analysis of increasingly realistic models
2. validation via human subject experiments on larger networks
3. **Outreach/collaboration for control community with**
   sociologists, psychologists, organization scientists
   on problems from Mathematical Sociology and Network Science