Opinion Dynamics and the Evolution of Influence Networks

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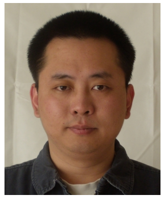
2014 SoCal Symp on Network Economics and Game Theory



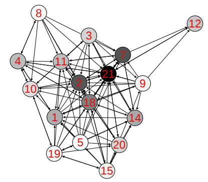
Peng Jia

Ana MirTabatabaei

Noah Friedkin

Motivations

- drivers
  - "big data" increasingle available
  - quantitative methods in social sciences
  - applications in marketing and (in)-security
- dynamical processes over social networks
  - opinion dynamics, info propagation
  - network formation and evolution
  - co-evolutionary processes
- key novelty: sequence of issues



Krackhardt's advice network

Small deliberative groups

- small deliberative groups are assembled in most social organization to deal with sequences of issues in particular domains:
  - judicial, legislative and executive branches: grand juries, federal panels of judges, Supreme Court – standing policy bodies, congressional committees – advisory boards
  - corporations: board of directors/trustees
  - universities: faculty meetings
- group properties may evolve over its issue sequence according to natural social processes that modify its internal social structure
- possible systematic changes:
  1. a stabilization of individuals' levels of openness and closure to interpersonal influences on their initial preferences,
  2. a stabilization of individuals' ranking of, and influence accorded to, other members'

Opinions, influence networks and centrality

Dynamics and Formation of Opinions

- convex combinations of opinions
- model by French ('56), Harary ('65), and DeGroot ('74)

Dynamics of Influence Networks and Social Power

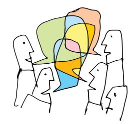
- reflected appraisal hypothesis by Cooley, 1902

  individual' self-appraisal (e.g., self-confidence, self-esteem, self-worth) is influenced by the appraisal of other individuals of her

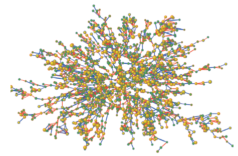
- mathematization by Friedkin, 2012:
  - varying social power and self-confidence
  - constant relative interpersonal relations

Network centrality

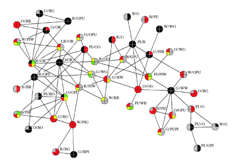
- centrality measure of network nodes, e.g., eigenvector centrality by Bonacich, 1972



Opinion formation



Social network for obesity study (Christakis and Fowler, 2007)



Social network for male wire-tailed manakins (Ryder et al. 2008)

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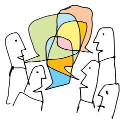

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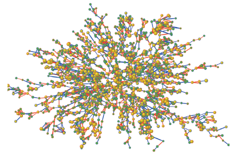

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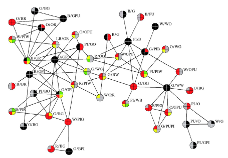

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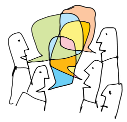

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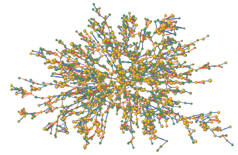

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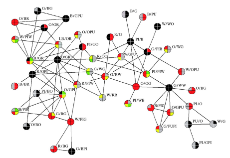

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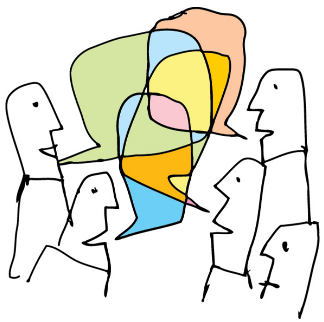
The dynamics of opinions

DeGroot opinion dynamics model

$$y(t+1) = W\,y(t)$$

- Opinions $y \in \mathbb{R}^n$
- Influence network = row-stochastic W
- by P-F: $\lim_{t\to\infty} y(t) = (w^T y(0))\mathbb{1}_n$ where w is dominant left eigenvector of W



- Self-weights $W_{ii} =: x_i$
- Interpersonal accorded weights W_{ij}
- Relative interpersonal accorded weights C_{ij}, where $W_{ij} = (1 - x_i)C_{ij}$

$$W(x) = \text{diag}(x)I_n + \text{diag}(\mathbb{1}_n - x)C$$

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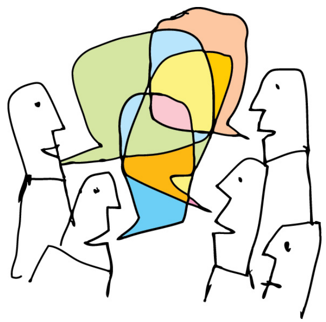

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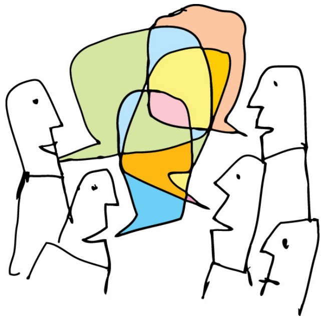

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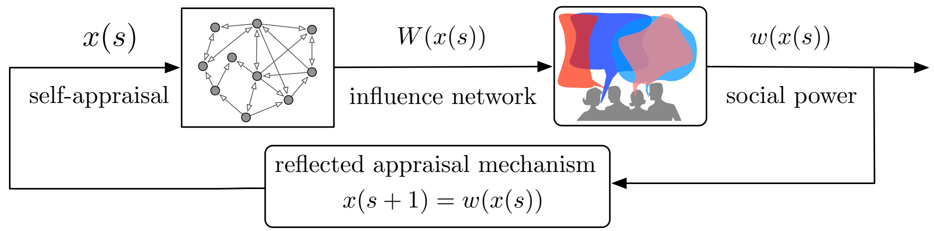

self-appraisal = self-weights ⟸ relative control = social power

The dynamical system

- DeGroot dynamics about an issue: $y(t+1) = W(x)y(t)$
- Influence network $W(x) = \mathrm{diag}(x)I_n + \mathrm{diag}(\mathbb{1}_n - x)C$
- Reflected appraisal across issues:

$$x(k+1) = w(x(k)) \qquad = F(x(k))$$

DeGroot-Friedkin dynamics

$$F(x) = \begin{cases} \mathbb{e}_i, & \text{if } x = \mathbb{e}_i \text{ for all } i \\ \left(\frac{c_1}{1-x_1}, \dots, \frac{c_n}{1-x_n}\right) \Big/ \sum_{i=1}^{n} \frac{c_i}{1-x_i}, & \text{otherwise} \end{cases}$$

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The map and the eigenvector centrality parameter

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- $F : \Delta_n \to \Delta_n$ locally Lipschitz
- The vertices $\{\mathbb{e}_i\}$ are fixed points under F
- relative interpersonal weights C play role only through c
- c = appropriate eigenvector centrality (dominant left eigenvector)

Lemma (Eigenvector centrality)

For any C row-stochastic, irreducible with zero diagonal and $c \in \Delta_n$,

- $\max\{c_i\} \leq 0.5$
- $c_i = 0.5 \iff$ *$G(C)$ is with star topology and i is the center*

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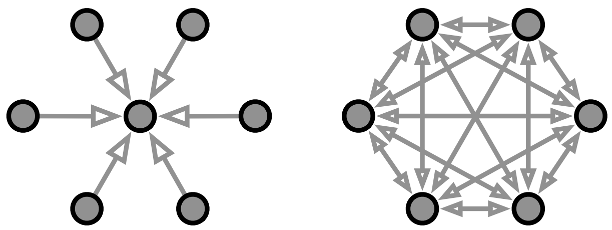
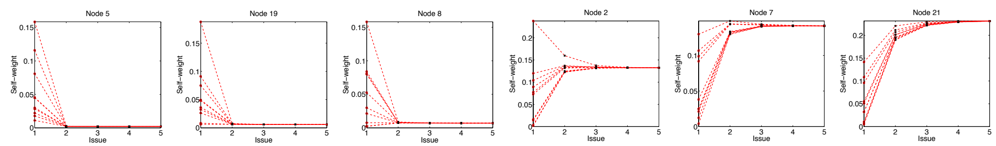
Problem: dynamical system analysis and sociological interpretation



- Existence and stability of equilibria for the D-F model?
- Role of network structure and parameters?
- Conditions of emergence of *autocracy* and *democracy*?
- Insight into "iron law of oligarchy" by Michels 1915?

Main results for generic "relative interpersonal accorded weights"

1. **unique non-trivial fixed point**: $x^* = x^*(c)$ in interior of Δ_n
2. **convergence = forgetting initial conditions**
   for all non-trivial initial conditions,
   $$\lim_{k\to\infty} x(k) = \lim_{k\to\infty} w(x(k)) = x^*$$
3. **accumulation of social power and self-appraisal**
   - fixed point $x^* > 0$ has same ordering of c
   - social power threshold T such that: $x_i^* \geq c_i \geq T$ or $x_i^* \leq c_i \leq T$



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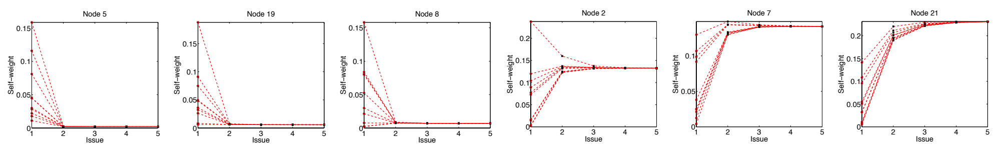

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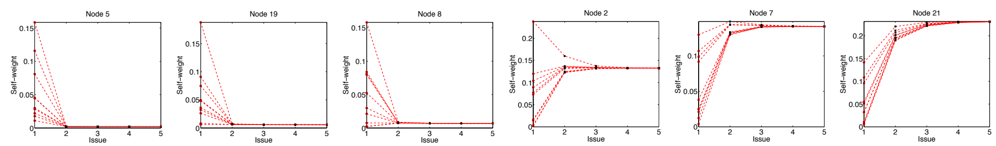
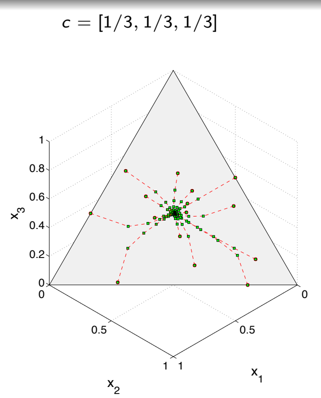


Doubly-stochastic C: emergency of democracy

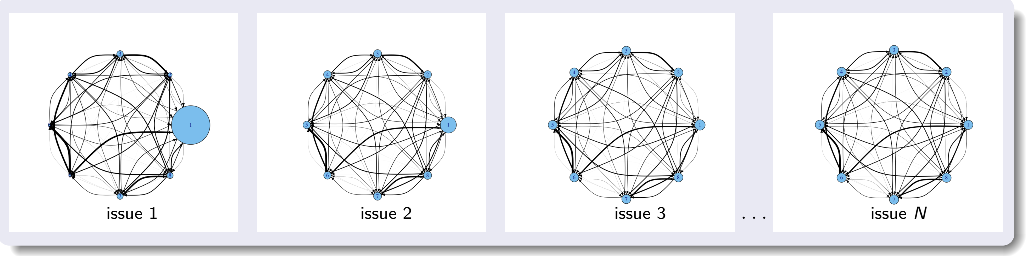
Lemma (Convergence to democracy)

Iff C is doubly-stochastic:

1. *the non-trivial fixed point of F is $\frac{\mathbb{1}_n}{n}$,*
2. *for all non-trivial initial conditions,* $\lim_{k\to\infty} x(k) = \lim_{k\to\infty} w(x(k)) = \frac{\mathbb{1}_n}{n}$.



- Uniform social power
- No power accumulation

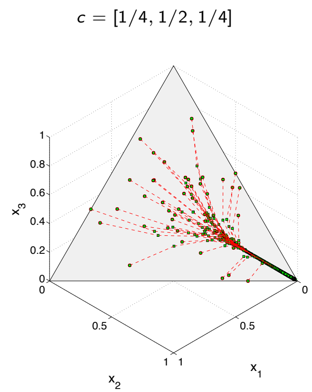


Star topology: emergency of autocracy

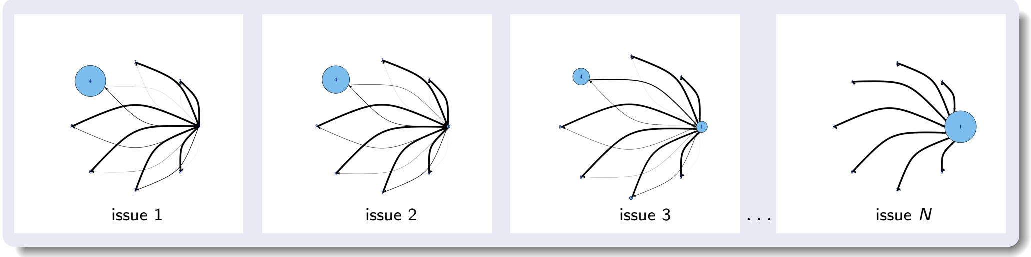
Lemma (Convergence to autocracy)

Iff graph has star topology with center j:

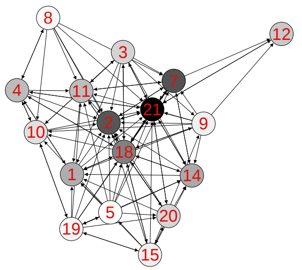
1. *there are no non-trivial fixed points of F*
2. *for all initial non-trivial conditions,* $\lim_{k\to\infty} x(k) = \lim_{s\to\infty} w(x(k)) = \mathbb{e}_j$.

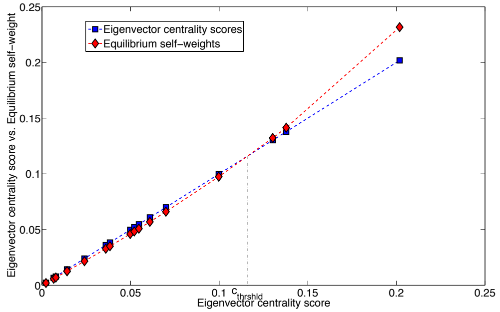


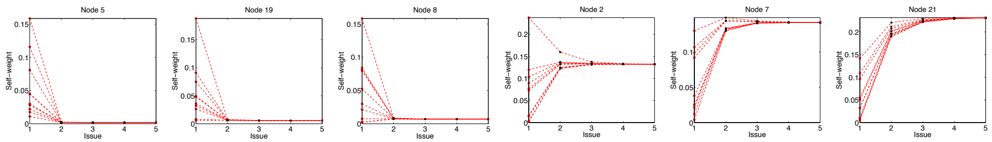
- Autocrat appears in star center
- Extreme power accumulation



D-F on Krackhardt's advice network







Proof methods

1. existence via Brower fixed point theorem (F continuous on compact)
2. ranking and uniqueness: elementary steps and contraddictions
3. monotonicity: $i_{\max}$ and $i_{\min}$ are invariant

$$i_{\max} = \operatorname{argmax}_j \frac{x_j(0)}{x_j^*} \implies i_{\max} = \operatorname{argmax}_j \frac{x_j(s)}{x_j^*}, \quad \forall s$$

4. convergence: Lyapunov function decreasing everywhere $x \neq x^*$

$$V(x) = \max_j \left(\ln \frac{x_j}{x_j^*}\right) - \min_j \left(\ln \frac{x_j}{x_j^*}\right)$$

Ongoing experiment

- 30 groups of 4 subjects in a face-to-face discussion
- opinion formation on a sequence of 15 issues
- issues in the domain of choice dilemmas:

  what is your minimum level of confidence (scored 0-100) required to accept a risky option with a high payoff rather than a less risky option with a low payoff

- 15 groups under pressure to reach consensus, other 15 no
- On each issue, each subject privately recorded (in following temporal order):
  1. an initial opinion on the issue prior to the group-discussion,
  2. a final opinion on the issue upon completion of the group-discussion (which ranged from 3-27 minutes), and
  3. an allocation of 100 influence units (under the instruction that these allocations should represent their appraisals of the relative influence of each group member's opinion on their own opinion).

Contributions and future work

Contributions

- a new perspective and a novel dynamical model
  for *social power, self-appraisal, influence networks*
- dynamics and feedback in sociology
- a new potential explanation for the emergence of autocracy
  see "iron law of oligarchy" by Michels 1911

Future work

- Robustness of results for distinct models of opinion dynamics
- Robustness of results for higher-order models of reflected appraisal

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