Geometry, Optimization and Control in Robot Coordination

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Coordination in multi-agent systems

What kind of systems?

- each agent senses its immediate environment,
- communicates with others,
- processes information gathered, and
- takes local action in response

"Raven" by AeroVironment Inc

"PackBot" by iRobot Inc

Wildebeest herd in the Serengeti

Geese flying in formation

Fish swarm in Atlantis aquarium

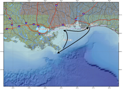
Cooperative robotics: technologies and applications

What scenarios?

Security systems, disaster recovery, environmental monitoring, study of natural phenomena and biological species, science imaging

Security systems

Warehouse automation



Environmental monitoring

What kind of tasks?

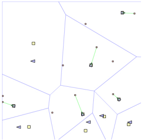
1. coordinated motion: rendezvous, flocking, formation
2. cooperative sensing: surveillance, exploration, search and rescue
3. cooperative material handling and transportation

Coordination via task and territory partitioning

Model: customers appear randomly in space/time
robotic network knows locations and provides service

Goal: minimize customer delay

Approach: assign customers to robots by partioning the space



F. Bullo, E. Frazzoli, M. Pavone, K. Savla, and S. L. Smith. Dynamic vehicle routing for robotic systems. *IEEE Proceedings*, May 2011. To appear

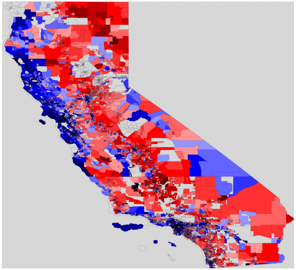
Outline

1. robot coordination via territory partitioning
2. gossip algorithms: mathematical setup
3. gossip algorithms: technological advances

Territory partitioning is ... art

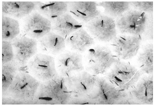
abstract expressionism
"Ocean Park No. 27" and "Ocean Park No. 129"
by Richard Diebenkorn (1922-1993), inspired by aerial landscapes

Territory partitioning ... centralized district design

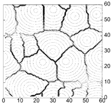


California Voting Districts: 2008 Obama/McCain votes

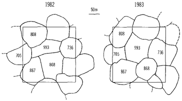
Territory partitioning is ... animal territory dynamics



Tilapia mossambica, "Hexagonal Territories," Barlow et al, '74



Red harvester ants, "Optimization, Conflict, and Nonoverlapping Foraging Ranges," Adler et al, '03



Sage sparrows, "Territory dynamics in a sage sparrows population," Petersen et al '87

Territory partitioning: behaviors and optimality

ANALYSIS of cooperative distributed behaviors

1. how do animals share territory?
   how do they decide foraging ranges?
   how do they decide nest locations?
2. what if each robot goes to "center" of own dominance region?
3. what if each robot moves away from closest robot?

DESIGN of performance metrics

4. how to cover a region with n minimum-radius overlapping disks?
5. how to design a minimum-distortion (fixed-rate) vector quantizer?
6. where to place mailboxes in a city / cache servers on the internet?

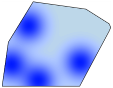
Multi-center functions

Expected wait time

$$H(p, v) = \int_{v_1} \|q - p_1\| dq \; + \; \cdots \; + \; \int_{v_n} \|q - p_n\| dq$$

- n robots at $p = \{p_1, \dots, p_n\}$
- environment is partitioned into $v = \{v_1, \dots, v_n\}$

$$H(p, v) = \sum_{i=1}^{n} \int_{v_i} f(\|q - p_i\|)\phi(q) dq$$

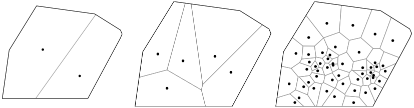


- $\phi : \mathbb{R}^2 \to \mathbb{R}_{\geq 0}$ density
- $f : \mathbb{R}_{\geq 0} \to \mathbb{R}$ penalty function

Optimal partitioning

The **Voronoi partition** $\{V_1, \dots, V_n\}$ generated by points $(p_1, \dots, p_n)$

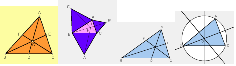
$$V_i = \{q \in Q \mid \; \|q - p_i\| \leq \|q - p_j\| \, , \; \forall j \neq i\}$$
$$= Q \bigcap_j (\text{half plane between } i \text{ and } j, \text{ containing } i)$$



Optimal centering (for region v with density ϕ)

function of p	minimizer = center
$p \mapsto \int_v \|q - p\|^2 \phi(q) dq$	**centroid** (or **center of mass**)
$p \mapsto \int_v \|q - p\| \phi(q) dq$	**Fermat–Weber point** (or **median**)
$p \mapsto \text{area}(v \cap \text{disk}(p, r))$	**r-area center**
$p \mapsto$ radius of largest disk centered at p enclosed inside v	**incenter**
$p \mapsto$ radius of smallest disk centered at p enclosing v	**circumcenter**

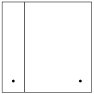
From online Encyclopedia of Triangle Centers



From optimality conditions to algorithms

$$H(p, v) = \int_{v_1} f(\|q - p_1\|)\phi(q)dq \; + \; \cdots \; + \; \int_{v_n} f(\|q - p_n\|)\phi(q)dq$$

1. at fixed positions, optimal partition is Voronoi
2. at fixed partition, optimal positions are "generalized centers"
3.



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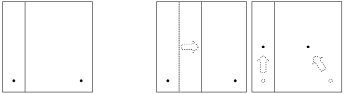
1. at fixed positions, optimal partition is Voronoi
2. at fixed partition, optimal positions are "generalized centers"
3. alternate v-p optimization $\implies$ local opt $=$ center Voronoi partition



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Voronoi+centering algorithm for robots

Voronoi+centering law

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At each comm round:
 1: acquire neighbors' positions
 2: compute own dominance region
 3: move towards center of own
    dominance region
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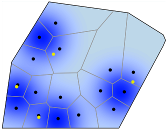
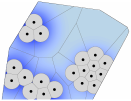
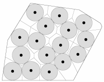
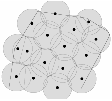




Area-center



Incenter



Circumcenter

F. Bullo, J. Cortés, and S. Martínez. *Distributed Control of Robotic Networks*. Applied Mathematics Series. Princeton Univ Press, 2009. Available at http://www.coordinationbook.info

Incomplete literature

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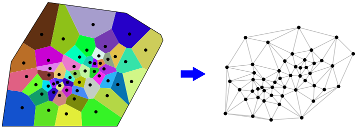
Outline

1. robot coordination via territory partitioning
2. gossip algorithms: mathematical setup
3. gossip algorithms: technological advances

Partitioning with minimal communication requirements

Voronoi+centering law requires:

1. synchronous communication
2. communication along edges of dual graph



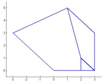
Minimalist coordination

- is synchrony necessary?
- what are minimal communication requirements?
- is asynchronous peer-to-peer, gossip, sufficient?

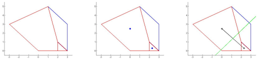
Gossip partitioning policy

At random comm instants, between two random regions:

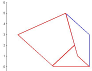
1. compute two centers
2. compute bisector of centers
3. partition two regions by bisector



before meeting



after meeting



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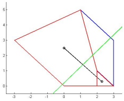
Gossip convergence analysis (proof sketch 1/4)

Lyapunov function for gossip territory partitioning

$$H(v) = \sum_{i=1}^{n} \int_{v_i} f(\| \mathrm{center}(v_i) - q \|) \phi(q) dq$$



1. state space is not finite-dimensional
   non-convex disconnected polygons
   arbitrary number of vertices
2. gossip map is not deterministic, ill-defined and discontinuous
   two regions could have same centers

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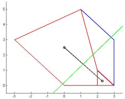

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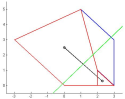

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Convergence with persistent switches (proof sketch 2/4)

- X is metric space
- finite collection of maps $T_i : X \to X$ for $i \in I$
- consider sequences $\{x_\ell\}_{\ell \geq 0} \subset X$ with

$$x_{\ell+1} = T_{i(\ell)}(x_\ell)$$

Assume:

1.
2.
3.
4.

If $x_0 \in W$, then almost surely

$$x_\ell \to \text{(intersection of sets of fixed points of all } T_i)$$

Convergence with persistent switches (proof sketch 2/4)

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$$x_{\ell+1} = T_{i(\ell)}(x_\ell)$$

Assume:

1. $W \subset X$ compact and positively invariant for each T_i
2. $U : W \to \mathbb{R}$ decreasing along each T_i
3. U and T_i are continuous on W
4. there exists probability $p \in]0, 1[$ such that, for all indices $i \in I$ and times ℓ, we have $\text{Prob}\left[x_{\ell+1} = T_i(x_\ell) \,|\, \text{past}\right] \geq p$

If $x_0 \in W$, then almost surely

$$x_\ell \to \text{(intersection of sets of fixed points of all } T_i)$$

The space of partitions (proof sketch 3/4)

Let $\mathcal{C}$ be set of closed subsets of Q — is it compact?

Hausdorff metric

$$d_{\mathrm{H}}(A,B) = \max\left\{\max_{a\in A}\min_{b\in B} d(a,b),\ \max_{b\in B}\min_{a\in A} d(a,b)\right\}$$

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1. $(\mathcal{C}, d_{\mathrm{H}})$ is compact metric space
2. dynamical system and Lyapunov function are not continuous wrt d_{H}!
3. Hausdorff metric sensitive to sets of measure zero

The space of partitions (proof sketch 4/4)

Let $\mathcal{C}$ be set of closed subsets of Q — is it compact?

Symmetric difference metric

$$d_{\mathrm{symm}}(A,B) = \mathrm{measure}(A\setminus B) + \mathrm{measure}(B\setminus A)$$

1.
2.
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The space of partitions (proof sketch 4/4)

Let $\mathcal{C}$ be set of closed subsets of Q — is it compact?

Symmetric difference metric

$$d_{\mathrm{symm}}(A,B) = \mathrm{measure}(A\setminus B) + \mathrm{measure}(B\setminus A)$$

1. redefine $\mathcal{C} \leftarrow \mathcal{C}/\sim$ where $A \sim B$ whenever $d_{\mathrm{symm}}(A,B) = 0$
2. dynamical system and Lyapunov function are continuous in $(\mathcal{C}, d_{\mathrm{symm}})$
3. no compactness result is available for $(\mathcal{C}, d_{\mathrm{symm}})$!
   Theorem: for any k, $(\mathcal{C}_{(k)}, d_{\mathrm{symm}})$ is compact.
   $\mathcal{C}_{(k)}$ is set of k-convex subsets (union of k convex sets)

Incomplete literature

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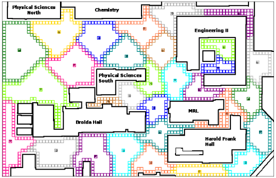
Gossip algorithms: technological advances



1. non-convex environments
2. motion protocols (for communication persistency)
3. hardware and large-scale implementations

Nonconvex environments as graphs

Revised setup

environment: weighted graph partitioned in connected subgraphs

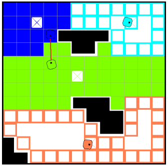
multi-center cost function: $H(p, v) = H_1(p_1, v_1) + \cdots + H_1(p_n, v_n)$

single-region cost function: $H_1(p, v) = \sum_{q \in v} \operatorname{dist}(p, q)\, \phi(q)$

center of subgraph v: minimizer of $p \mapsto H_1(p, v)$

Range-dependent stochastic comm

Two robots communicate at the sample times of a Poisson process with distance-dependent intensity



Nonconvex environments as graphs

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- environment: weighted graph partitioned in connected subgraphs
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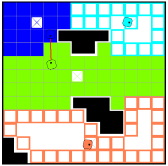


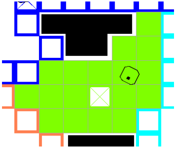
Discretized gossip algorithm

1. Ensure that neighbors meet frequently enough:
   $\Rightarrow$ Random Destination & Wait Motion Protocol
2. Update partition when two robots meet:
   $\Rightarrow$ Pairwise Partitioning Rule

Random Destination & Wait Motion Protocol

Each robot continuously executes:

1: select sample destination $q_i \in v_i$
2: move to q_i
3: wait at q_i for time $\tau > 0$



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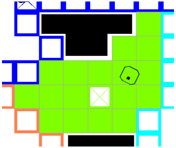

Discretized gossip algorithm

Pairwise Partitioning Rule

Whenever robots i and j communicate:

$w \leftarrow v_i \cup v_j$
while (computation time is available) **do**
  $(q_i, q_j) \leftarrow$ sample vertices in w
  $(w_i, w_j) \leftarrow$ Voronoi of w by (q_i, q_j)
  if $(H_1(q_i, w_i) + H_1(q_j, w_j))$ improves) **then**
    centroids $\leftarrow (q_i, q_j)$
    $(v_i, v_j) \leftarrow (w_i, w_j)$
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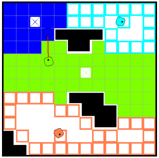

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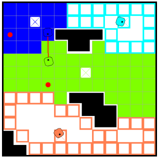

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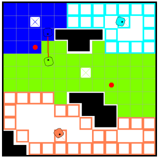
end while



J. W. Durham, R. Carli, P. Frasca, and F. Bullo. Discrete partitioning and coverage control for gossiping robots. *IEEE Trans Robotics*, November 2010. Submitted

Discretized gossip algorithm

Pairwise Partitioning Rule

Whenever robots i and j communicate:

$w \leftarrow v_i \cup v_j$
while (computation time is available) **do**
  $(q_i, q_j) \leftarrow$ sample vertices in w
  $(w_i, w_j) \leftarrow$ Voronoi of w by (q_i, q_j)
  if $(H_1(q_i, w_i) + H_1(q_j, w_j))$ improves) **then**
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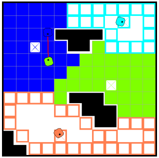

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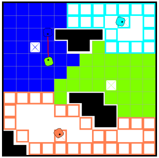


(combinatorial optimization) – interruptible anytime algorithm

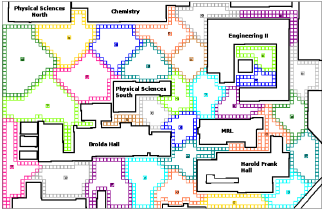
J. W. Durham, R. Carli, P. Frasca, and F. Bullo. Discrete partitioning and coverage control for gossiping robots. *IEEE Trans Robotics*, November 2010. Submitted

Harware-in-the-loop experiment

Player/Stage robot simulation & control system: realistic robot models with integrated wireless network model & obstacle-avoidance planner.

Hardware-in-the-loop experiment: 3 physical and 6 simulated robots

Larger-scale simulation experiment



Player/Stage robot simulation & control system: realistic robot models with integrated wireless network model & obstacle-avoidance planner.

Simulation experiment: 30 robots; UCSB campus.

Conclusions

Summary

1. gossip algorithms: mathematical setup
2. gossip algorithms: technological advances

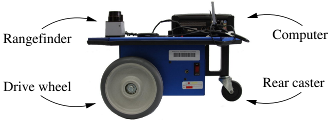
Open problems

1. topology and comp geometry of power sets
2. coordination: resource allocation, weak comm protocols
3. ecology of territory partioning

Acknowledgements

- Ruggero Carli, Assistant Professor @ Universita di Padova
- Paolo Frasca, Postdoc @ Politecnico di Torino
- Joey W. Durham, Senior Engineer @ Kiva Systems
- generous support from NSF, ARO, ONR, AFOSR

Robot hardware



Localization:
Adaptive Monte Carlo Localization particle filter method for matching scans to a map

(Thrun et al., 2001)

Navigation:
Smooth Nearness Diagram navigation for local obstacle avoidance using sensor data

(Durham et al., 2008)

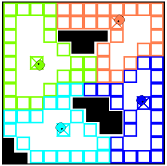
Convergence Result

Theorem

Convergence almost surely to a pairwise-optimal partition in finite time.

Proof sketch

1. Algorithm maintains a connected n-partition
2. Probability neighbors communicate in any interval
3. H decreases with every pairwise update
4. Pairwise-optimal partitions are equilibrium set



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Pairwise Optimal Partitions

Definition

A partition is pairwise-optimal if every pair of neighboring robots (i, j) has reached lowest possible coverage cost of $v_i \cup v_j$, i.e. that

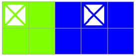
$$H_1(c_i; v_i) + H_1(c_j; v_j) = \min_{a,b \in w} \left\{ \sum_{k \in w} \min\left\{ d_w(a,k), d_w(b,k) \right\} \right\}$$

where $w = v_i \cup v_j$ and (c_i, c_j) are the centers of (v_i, v_j)

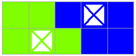
$\Rightarrow$ Every pairwise-optimal partition is also centroidal Voronoi

Subset Example

Cost: 12 hops



Cost: 11 hops

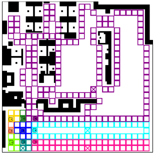


Cost: 10 hops

- All are centroidal Voronoi partitions
- Only lowest cost is pairwise-optimal (by definition)

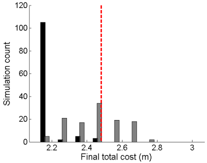
$\Rightarrow$ Avoid all pairwise local minima

Monte Carlo Results I



Initial cost: 5.48 *m*
Optimum cost: 2.17 *m*

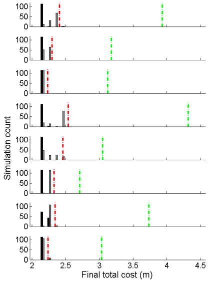
116 sequences of random pairwise exchanges



Black - Pairwise-optimal Algorithm
Gray - Gossip Lloyd Algorithm
Red - Lloyd Algorithm

$\Rightarrow$ 99% confidence that with at least 80% probability the Pairwise-optimal algorithm gets within 4% of the global optimum

Monte Carlo Results II

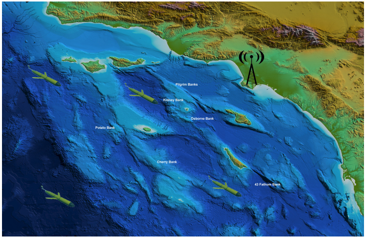


Tests shown:

- 8 random initial conditions
- 116 sequences of pairwise exchanges

Black - Pairwise-optimal Algorithm
Gray - Gossip Lloyd Algorithm
Red - Lloyd Algorithm
Green - Starting cost

Motivational Scenario



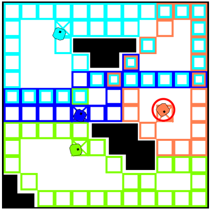
One-to-Base Communication Model

For each robot, there exists a finite upper bound Δ on the time between its communications with the base station.

Changes from Gossip Case

New Approach:

- One-sided territory & centroid updates
- Evolve overlapping territories



New cost function

$$H_{\max}(c, v) = \sum_{q \in Q} \max_{i \in \{1,\dots,n\}} \{\mathrm{dist}(c_i, a) \mid q \in v_i\} \, \phi(q)$$

One-to-Base Partitioning

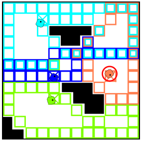
One-to-Base Partitioning

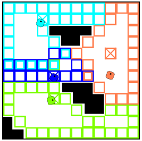
Base station holds local copy of robot territories

When robot i talks to base:

1: Update robot i's centroid
2: Transmit local copy of v_i to robot i
3: **for** every other robot j **do**
4: Add vertices to v_j which are in v_i but closer to j
5: Remove vertices from v_j which are in both but closer to i
6: **end for**

$\Rightarrow$ Split centering and partitioning





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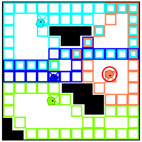
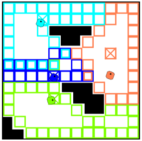




Convergence Results

Theorem (Durham et al., 2011)

Convergence to a centroidal Voronoi partition in finite time.

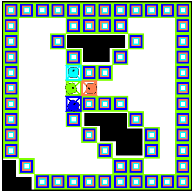
Let $M(P)$ be set of vertices owned by multiple robots and

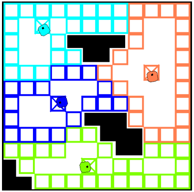
$$H_{\min}(c, v) = \sum_{q \in Q} \min_{i \in \{1,\dots,n\}} \{\mathrm{dist}(c_i, q) \mid q \in v_i\} \, \phi(q)$$

Proof of Decreasing Cost: One of these conditions holds

1. $H_{\max}(c^+, v^+) < H_{\max}(c, v)$
2. $H_{\max}(c^+, v^+) = H_{\max}(c, v)$ and $H_{\min}(c^+, v^+) < H_{\min}(c, P)$
3. $H_{\max}(c^+, v^+) = H_{\max}(c, v)$, $H_{\min}(c^+, v^+) = H_{\min}(c, v)$, and $|M(v^+)| < |M(v)|$

Simulation Movie





Four robots each initially own the entire environment, but then settle on a centroidal Voronoi partition