Geometry, Optimization and Control in Robot Coordination

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- Jorge Cortés (UCSD): robotic networks, multi-center optimization, nonconvex deployment and rendezvous
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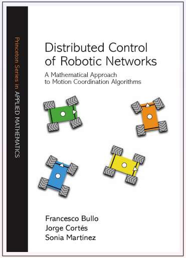
- Karl J. Obermeyer (UCSB): nonconvex deployment

Distributed Control of Robotic Networks



1. intro to distributed algorithms (graph theory, synchronous networks, and averaging algos)
2. geometric models and geometric optimization problems
3. model for robotic, relative sensing networks, and complexity
4. algorithms for rendezvous, deployment, boundary estimation

Manuscript by F. Bullo, J. Cortés, and S. Martínez. Princeton Univ Press, 2009, ISBN 978-0-691-14195-4. Freely downloadable at http://coordinationbook.info with tutorial slides and (ongoing) software libraries.

Cooperative multi-agent systems

What kind of systems?
Groups of agents with control, sensing, communication and computing

What kind of abilities?

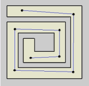
- each agent senses its immediate environment,
- communicates with others,
- processes information gathered, and
- takes local action in response

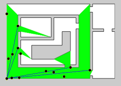
AeroVironment Inc, “Raven” unmanned aerial vehicle

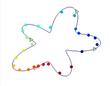
iRobot Inc, “PackBot” unmanned ground vehicle

Cooperative systems: technologies and applications

What kind of tasks?





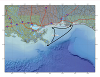


What scenarios?

Security systems, disaster recovery, environmental monitoring, study of natural phenomena and biological species, science imaging

Security systems

Building monitoring and evac



Environmental monitoring

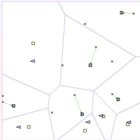
Today's outline

1. **vehicle routing problems**
   via queueing theory and combinatorics
2. **territory partitioning**
   via emerging behaviors and geometric optimization
3. **peer-to-peer coordination**
   via invariance principle on metric space

Queueing theory for robotic networks

Dynamic Vehicle Routing

- customers appear randomly space/time
- robotic network knows locations and provides service
- Goal: distributed adaptive algos, delay vs throughput



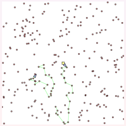
M. Pavone, E. Frazzoli, and F. Bullo. Decentralized algorithms for stochastic and dynamic vehicle routing with general target distribution. In *IEEE Conf. on Decision and Control*, pages 4869–4874, New Orleans, LA, December 2007

Algo #1: Receding-horizon shortest-path policy

Receding-horizon Shortest-Path (RH-SP)

For $\eta \in (0, 1]$, single agent performs:

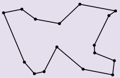
```
while no customers, move to center
while customers waiting
   1 compute shortest path through current
     customers
   2 service η-fraction of path
```



- shortest path is NP-hard, but effective heuristics available
- delay is optimal in light traffic
- delay is constant-factor optimal in high traffic

Algo #1: Sketch of RH-SP analysis via combinatorics in Euclidean space

1. queue is stable if service time < interarrival time
2. service time $= \dfrac{\text{length shortest path}(n)}{n}$ $(n = \#\text{ customers})$
3. queue is stable if (length of shortest path) = sublinear f(n)



length shortest path$(n) \sim \sqrt{n}$

J. Beardwood, J. Halton, and J. Hammersly. The shortest path through many points. In *Proceedings of the Cambridge Philosophy Society*, volume 55, pages 299–327, 1959

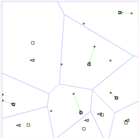
Algo #2: Load balancing via territory partitioning

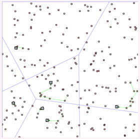
RH-SP + Partitioning

For $\eta \in (0, 1]$, agent i performs:

1: compute own cell v_i in optimal partition
2: apply RH-SP policy on v_i

Asymptotically constant-factor optimal in light and high traffic



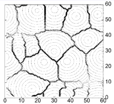


Outline

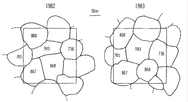
1. **vehicle routing problems**
   via queueing theory and combinatorics
2. **territory partitioning**
   via emerging behaviors and geometric optimization
3. **peer-to-peer coordination**
   via invariance principle on metric space

Territory partitioning akin to *animal territory dynamics*

Tilapia mossambica, "Hexagonal Territories," Barlow et al, '74



Red harvester ants, "Optimization, Conflict, and Nonoverlapping Foraging Ranges," Adler et al, '03

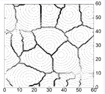


Sage sparrows, "Territory dynamics in a sage sparrows population," Petersen et al '87

Territory partitioning: behaviors and optimality

ANALYSIS of cooperative distributed behaviors

1. how do animals share territory?
   how do they decide foraging ranges?
   how do they decide nest locations?
2. what if each robot goes to "center" of own dominance region?
3. what if each robot moves away from closest vehicle?



DESIGN of performance metrics

4. how to cover a region with n minimum-radius overlapping disks?
5. how to design a minimum-distortion (fixed-rate) vector quantizer?
6. where to place mailboxes in a city / cache servers on the internet?

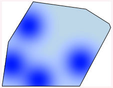
Multi-center functions

Expected wait time

$$H(p, v) = \int_{v_1} \|q - p_1\| dq + \cdots + \int_{v_n} \|q - p_n\| dq$$

- n robots at $p = \{p_1, \ldots, p_n\}$
- environment is partitioned into $v = \{v_1, \ldots, v_n\}$

$$H(p, v) = \sum_{i=1}^{n} \int_{v_i} f(\|q - p_i\|)\phi(q) dq$$

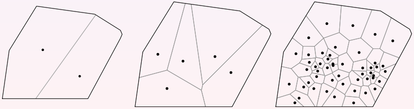


- $\phi : \mathbb{R}^2 \to \mathbb{R}_{\geq 0}$ density
- $f : \mathbb{R}_{\geq 0} \to \mathbb{R}$ penalty function

Optimal partitioning by Georgy Fedoseevich Voronoy (PhD from Saint Petersburg State University in 1896)

The Voronoi partition $\{V_1, \ldots, V_n\}$ generated by points $(p_1, \ldots, p_n)$

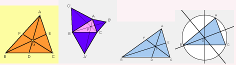
$$V_i = \{q \in Q \mid \|q - p_i\| \leq \|q - p_j\|, \ \forall j \neq i\}$$
$$= Q \bigcap_j (\text{half plane between } i \text{ and } j, \text{ containing } i)$$



Optimal centering (for region v with density ϕ)

function of p	minimizer = center
$p \mapsto \int_v \|q - p\|^2 \phi(q) dq$	centroid (or center of mass)
$p \mapsto \int_v \|q - p\| \phi(q) dq$	Fermat–Weber point (or median)
$p \mapsto \text{area}(v \cap \text{disk}(p, r))$	r-area center
$p \mapsto$ radius of largest disk centered at p enclosed inside v	incenter
$p \mapsto$ radius of smallest disk centered at p enclosing v	circumcenter

From online Encyclopedia of Triangle Centers



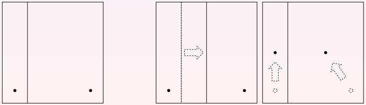
From optimality conditions to algorithms

$$H(p, v) = \sum_{i=1}^{n} \int_{v_i} f(\|q - p_i\|)\phi(q)dq$$

Theorem (Alternating Algorithm, Lloyd '57)

1. *at fixed positions, optimal partition is Voronoi*
2. *at fixed partition, optimal positions are "generalized centers"*
3. *alternate v-p optimization*

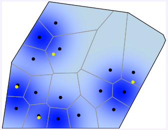
$\implies$ *local optimum = center Voronoi partition*

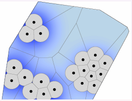


Voronoi+centering algorithm

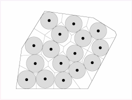
Voronoi+centering law

```
At each comm round:
 1: acquire neighbors' positions
 2: compute own dominance region
 3: move towards center of own
    dominance region
```

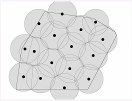




Area-center



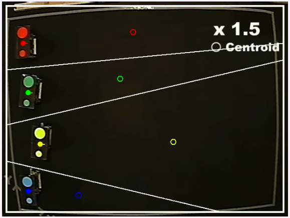
Incenter



Circumcenter

J. Cortés, S. Martínez, and F. Bullo. Spatially-distributed coverage optimization and control with limited-range interactions. *ESAIM: Control, Optimisation & Calculus of Variations*, 11:691–719, 2005

Experimental Territory Partitioning



Takahide Goto, Takeshi Hatanaka, Masayuki Fujita

Tokyo Institute of Technology

Experimental Territory Partitioning

Mac Schwager, Brian Julian, Daniela Rus

Distributed Robots Laboratory, MIT

Outline

1. **vehicle routing problems**
   via queueing theory and combinatorics

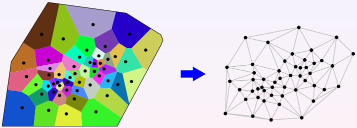
2. **territory partitioning**
   via emerging behaviors and geometric optimization

3. **peer-to-peer coordination**
   via invariance principle on metric space

Partitioning with minimal communication requirements

Voronoi+centering law requires:

1. synchronous communication
2. communication along edges of dual graph

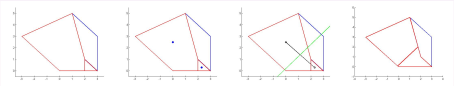


Minimalist coordination

- is synchrony necessary?
- is it sufficient to communicate peer-to-peer (gossip)?
- what are minimal requirements?

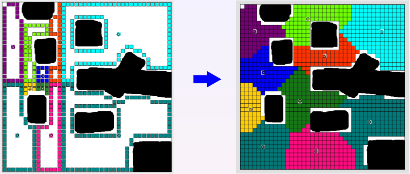
Peer-to-peer partitioning policy

1. Random communication between two regions
2. Compute two centers
3. Compute bisector of centers
4. Partition two regions by bisector



P. Frasca, R. Carli, and F. Bullo. Multiagent coverage algorithms with gossip communication: control systems on the space of partitions. In *American Control Conference*, pages 2228–2235, St. Louis, MO, June 2009

Indoor example implementation



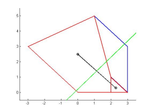
- Player/Stage platform
- realistic robot models in discretized environments
- integrated wireless network model & obstacle-avoidance planner

J. W. Durham, R. Carli, P. Frasca, and F. Bullo. Discrete partitioning and coverage control with gossip communication. In *ASME Dynamic Systems and Control Conference*, Hollywood, CA, October 2009. to appear

Peer-to-peer convergence analysis (proof sketch 1/3)

Lyapunov function for peer-to-peer territory partitioning

$$H(v) = \sum_{i=1}^{n} \int_{v_i} f(\| \operatorname{center}(v_i) - q \|) \phi(q) dq$$



1. state space is not finite-dimensional
   - non-convex disconnected polygons
   - arbitrary number of vertices
2. peer-to-peer map is not deterministic, ill-defined and discontinuous
   - two regions could have same centers

The space of partitions (proof sketch 2/3)

Definition (space of n-partitions)

v is collections of n subsets of Q, $\{v_1, \dots, v_n\}$, such that

1. $v_1 \cup \dots \cup v_n = Q$,
2. $\operatorname{interior}(v_i) \cap \operatorname{interior}(v_j) = \emptyset$ if $i \neq j$, and
3. each v_i is closed, has non-empty interior and zero-measure boundary

Given sets A, B, symmetric difference and distance are:

$$d_\Delta(A, B) = \text{area}\Big((\text{points in } A \text{ that are not in } B) \cup (\text{vice versa})\Big)$$

Theorem (topological properties of the space of partitions)

Partition space with $(u, v) \mapsto \sum_{i=1}^{n} d_\Delta(u_i, v_i)$ is metric and precompact

Convergence with persistent switches (proof sketch 3/3)

- X is metric space
- finite collection of maps $T_i : X \to X$ for $i \in I$
- consider sequences $\{x_\ell\}_{\ell \geq 0} \subset X$ with

$$x_{\ell+1} = T_{i(\ell)}(x_\ell)$$

Assume:

1. $W \subset X$ compact and positively invariant for each T_i
2. $U : W \to \mathbb{R}$ decreasing along each T_i
3. U and T_i are continuous on W
4. there exists probability $p \in]0, 1[$ such that, for all indices $i \in I$ and times ℓ, we have $\operatorname{Prob}\left[x_{\ell+1} = T_i(x_\ell) \,|\, \text{past}\right] \geq p$

If $x_0 \in W$, then almost surely

$$x_\ell \to (\text{intersection of sets of fixed points of all } T_i) \cap U^{-1}(c)$$

Emerging discipline: robotic networks

Robotic Network Theory

1. network modeling
   - network, ctrl+comm algorithm, task, complexity
2. coordination algorithm
   - partitioning, vehicle routing, task allocation

Open problems

1. algorithmic design for minimalist robotic networks
   - scalable, adaptive, asynchronous, agent arrival/departure
   - rich task set, e.g., cooperative estimation
2. mixed robotic-human networks
3. high-fidelity sensing/actuation scenarios