Deployment and Territory Partitioning for Gossiping Robots

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Minimalist robots and motion coordination

What kind of systems?
Groups of systems with control, sensing, communication and computing

Individual members in the group can

- sense its immediate environment
- communicate with others
- process the information gathered
- take a local action in response

Wildebeest herd in the Serengeti

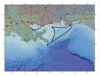
Geese flying in formation

Atlantis aquarium, CDC 2004

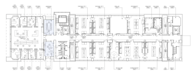
Minimalist robots: technologies and applications

AeroVironment Inc, "Raven"
small unmanned aerial vehicle

iRobot Inc, "PackBot"
unmanned ground vehicle



Environmental monitoring



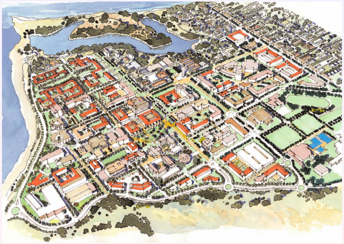
Building monitoring and evac

Security systems

Territory partitioning is ... art

Ocean Park Paintings, by Richard Diebenkorn (1922-1993)

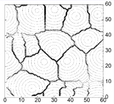
Territory partitioning is ... centralized space allocation



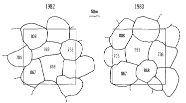
UCSB Campus Development Plan, 2008

Territory partitioning is ... animal territory dynamics

Tilapia mossambica, "Hexagonal Territories," Barlow et al, '74



Red harvester ants, "Optimization, Conflict, and Nonoverlapping Foraging Ranges," Adler et al, '03

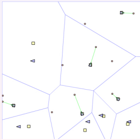


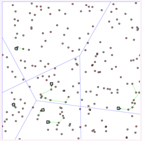
Sage sparrows, "Territory dynamics in a sage sparrows population," Petersen et al '87

Territory partitioning is ... robotic load balancing

Dynamic Vehicle Routing

- targets/customers appear randomly space/time
- robotic network knows locations and provides service
- Goal: distributed adaptive algos, delay vs throughput



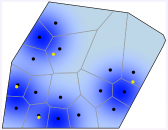


M. Pavone, E. Frazzoli, and F. Bullo. Decentralized algorithms for stochastic and dynamic vehicle routing with general target distribution. In *Proc CDC*, pages 4869–4874, New Orleans, LA, December 2007. URL `http://motion.me.ucsb.edu/pdf/2007g-pfb.pdf`

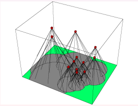
Distributed partitioning+centering algorithm

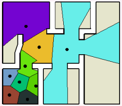
Partitioning+centering law

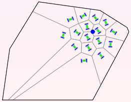
```
At each comm round:
 1: acquire neighbors' positions
 2: compute own dominance region
 3: move towards centroid of own
    dominance region
```



J. Cortés, S. Martínez, T. Karatas, and F. Bullo. Coverage control for mobile sensing networks. *IEEE Trans Robotics & Automation*, 20(2):243–255, 2004







Multi-center optimization

- take environment with density function $\phi : Q \to \mathbb{R}_{\geq 0}$
- place N robots at $p = \{p_1, \dots, p_N\}$
- partition environment into $v = \{v_1, \dots, v_N\}$
- define expected quadratic deviation

$$H(v,p) = \sum_{i=1}^{N} \int_{v_i} f(\|q - p_i\|)\phi(q)dq$$

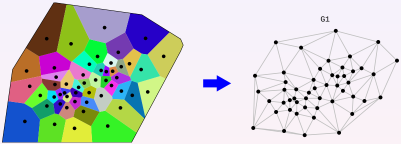
Theorem (Lloyd '57 "least-square quantization")

1. *at fixed partition, optimal positions are centroids*
2. *at fixed positions, optimal partition is Voronoi*
3. *Lloyd algorithm:*
   1. *alternate p-v optimization*
   2. *convergence to centroidal Voronoi partition*

Today: What are minimal communication requirements?

Partitioning+centering law requires:

1. synchronous communication
2. communication along edges of dual graph

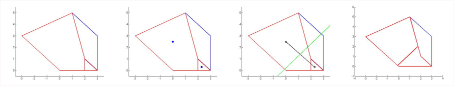


Minimalist robotics: what are minimal comm requirements?

- is synchrony necessary?
- is it sufficient to communicate peer-to-peer (gossip)?
- what are minimal requirements?

Gossip partitioning policy

1. Random communication between two regions
2. Compute two centers
3. Compute bisector of centers
4. Partition two regions by bisector



P. Frasca, R. Carli, and F. Bullo. Multiagent coverage algorithms with gossip communication: control systems on the space of partitions. In *Proc ACC*, pages 2228–2235, St. Louis, MO, June 2009

From standard to gossip algorithm

Standard partitioning+centering algorithm

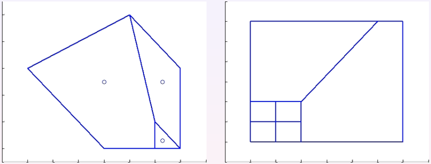
1. robot talks to all its neighbors in dual graph
2. robot computes its Voronoi region
3. robot moves to centroid of its Voronoi region

Gossip partitioning policy

1. robot/region talks to only one neighboring robot/region
2. two regions are updated according to

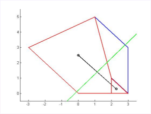
$$v_i^+ := \{q \in v_i \cup v_j \mid \|q - \mathrm{centroid}(v_i)\| \leq \|q - \mathrm{centroid}(v_j)\|\}$$

Simulation results



Implementation: centralized, General Polygon Clipper (GPC) library

Technical challenges



1. state space is not finite-dimensional
   - non-convex disconnected polygons
   - arbitrary number of vertices
2. gossip map is not deterministic, ill-defined and discontinuous
   - two regions could have same centroid
   - disconnected/connected discontinuity
3. Lyapunov function missing

4. **motion protocol for deterministic/random meetings**

From standard to Lyapunov functions for partitions

Standard coverage control

robot i moves towards centroid of its Voronoi region

$$H(p_1, \dots, p_N) = \sum_{i=1}^{N} \int_{V_i(p_1,\dots,p_N)} f(\|p_i - q\|)\phi(q)dq$$

Gossip coverage control

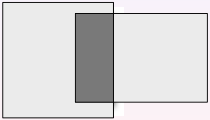
region v_i is modified to appear like a Voronoi region

$$H(v_1, \dots, v_N) = \sum_{i=1}^{N} \int_{v_i} f(\|\operatorname{centroid}(v_i) - q\|)\phi(q)dq$$

Symmetric difference

Given sets A, B, symmetric difference and distance are:

$$A\Delta B = (A \cup B) \setminus (A \cap B), \qquad d_\Delta(A, B) = \text{measure}(A\Delta B)$$



The space of partitions

Definition (space of N-partitions)

$\mathcal{V}_N$ is collections of N subsets of Q, $v = \{v_i\}_{i=1}^N$, such that

1. $v_i \neq \emptyset$ and $v_i = \overline{\text{interior}(v_i)}$
2. $\text{interior}(v_i) \cap \text{interior}(v_j) = \emptyset$ if $i \neq j$, and
3. $\bigcup_{i=1}^N v_i = Q$

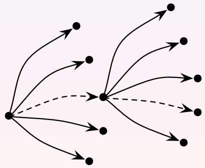
Theorem (topological properties of the space of partitions)

$\mathcal{V}_N$ with $d_\Delta(u, v) = \sum_{i=1}^N d_\Delta(u_i, v_i)$ is metric and precompact

Convergence theorem: nondeterminism

- X is metric space
- set-valued $T : X \rightrightarrows X$ with $T(x) = \{T_i(x)\}_{i \in I}$ for finite I
- consider sequences $\{x_n\}_{n \geq 0} \subset X$ with

$$x_{n+1} \in T(x_n)$$



Convergence thm: uniformly persistent switches

- X is metric space
- set-valued $T : X \rightrightarrows X$ with $T(x) = \{T_i(x)\}_{i \in I}$ for finite I
- consider a sequence $\{x_n\}_{n \geq 0} \subset X$ with

$$x_{n+1} \in T(x_n)$$

Assume:

1. $W \subset X$ compact and positively invariant for T
2. $U : W \to \mathbb{R}$ non-increasing along T, decreasing along $T \setminus \{\text{id}\}$
3. U and T_i are continuous on W
4. for all $i \in I$, there are infinite times n such that $x_{n+1} = T_i(x_n)$ and delay between any two consecutive times is bounded

If $x_0 \in W$, then

$$x_n \to \{x \in W \mid x = T_i(x) \text{ for all } i \in I\} \cap U^{-1}(c)$$

Convergence thm: randomly persistent switches

- X is metric space
- set-valued $T : X \rightrightarrows X$ with $T(x) = \{T_i(x)\}_{i \in I}$ for finite I
- consider sequences $\{x_n\}_{n \geq 0} \subset X$ with

$$x_{n+1} \in T(x_n)$$

Assume:

1. $W \subset X$ compact and positively invariant for T
2. $U : W \to \mathbb{R}$ non-increasing along T, decreasing along $T \setminus \{\text{id}\}$
3. U and T_i are continuous on W
4. there exists probability $p \in]0, 1[$ such that, for all indices $i \in I$ and times n, we have $\mathbb{P}\big[x_{n+1} = T_i(x_n) \,|\, \text{past}\big] \geq p$

If $x_0 \in W$, then almost surely

$$x_n \to \{x \in W \mid x = T_i(x) \text{ for all } i \in I\} \cap U^{-1}(c)$$

Conclusions

Summary

1. novel gossip partitioning algorithm
2. space of partitions
3. convergence theorem for switching maps
4. convergence to centroidal Voronoi partition

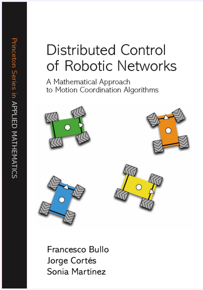
P. Frasca, R. Carli, and F. Bullo. Multiagent coverage algorithms with gossip communication: control systems on the space of partitions. In *Proc ACC*, pages 2228–2235, St. Louis, MO, June 2009

Ongoing work

1. motion laws to maximize peer-to-peer meeting frequencies
2. convergence rates: known in 1D; unknown in 2D
3. robots arriving/departing
4. more general version of partitioning:

   nonsmooth, equitable, nonconvex, 3D

"Distributed Control of Robotic Networks"



1. intro to distributed algorithms (graph theory, synchronous networks, and averaging algos)
2. geometric models and geometric optimization problems
3. model for robotic, relative sensing networks, and complexity
4. algorithms for rendezvous, deployment, boundary estimation

Status: Freely downloadable at `http://coordinationbook.info` with tutorial slides and (ongoing) software libraries.

Emerging discipline: robotic networks

- **network modeling**

  network, ctrl+comm algorithm, task, complexity
- **coordination algorithm**

  deployment, task allocation, boundary estimation

Open problems

1. algorithmic design for minimalist robotic networks

   scalable, adaptive, asynchronous, agent arrival/departure

   tasks: search, exploration, identify and track
2. integrated coordination, communication, and estimation
3. complex sensing/actuation scenarios