Boundary patrol using robotic networks without localization

Topology and Minimalism
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Incomplete state of the art

AeroVironment Inc, "Raven" small unmanned aerial vehicle

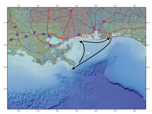
iRobot Inc, "PackBot" unmanned ground vehicle

Distributed algorithms
automata-theoretic: "Distributed Algorithms" by N. Lynch, D. Peleg
numerical: "Parallel and Distributed Computation" by by Bertsekas and Tsitsiklis

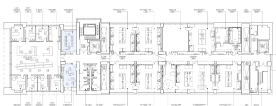
Motion coordination
"rendezvous" by Suzuki and Yamashita
"consensus, flocking, agreement" by Jadbabaie, Olfati-Saber
"formation control" by Baillieul, Morse, Anderson

Research directions

Build: distributed systems embedded actuator/sensors networks **Develop distributed disciplines:** (i) sensor fusion (ii) communications (iii) coordinated control (iv) task allocation and scheduling	**Challenges** (i) scalability (ii) performance (iii) robustness (iv) models



Environmental monitoring



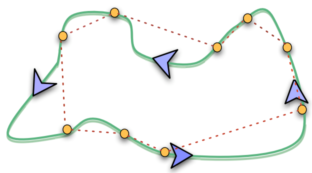
Building monitoring and evac

Security systems

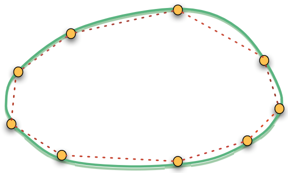
Scenario 1: Boundary estimation

Assumption: local sensing and tracking, interpolation via waypoints
Objective: estimate/interpolate moving boundary

adaptive polygonal approximation



Scenario 1: Interpolation theory

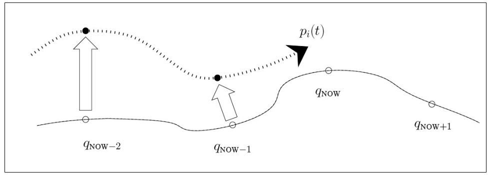


For strictly convex bodies (Gruber '80)

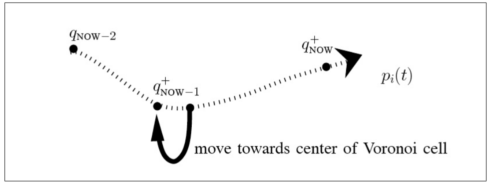
- **sufficient condition for optimality:** each two consecutive interpolation points p_k, p_{k+1} are separated by same line integral $\int_{p_k \to p_{k+1}} \kappa(\ell)^{1/3} d\ell$
- **error estimate** $\approx \frac{1}{12n^2} \left(\int_{\partial Q} \kappa(\ell)^{1/3} d\ell \right)^3$

Scenario 1: Estimate-Update and Pursuit

(i) **projection step**

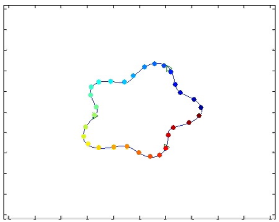


(ii) **update interpolation points** for "pseudo-uniform" interpolation placement



(iii) **accelerate/decelerate** for uniform vehicle placement

Scenario 1: Performance/robustness



(i) asynchronous distributed over ring

(ii) convergence to equally distributed interpolation points and equally spaced vehicles

(iii) time complexity: worst case $O(n^2 \log(n))$, where $n = \frac{\#\text{ interpolation points}}{\#\text{ vehicles}}$

(iv) ISS robust to: evolving boundary, interpolation, sensor noise

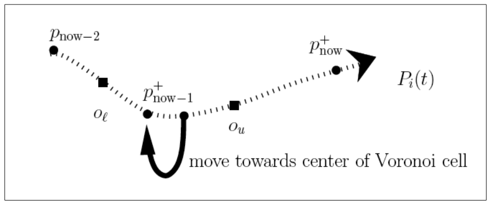
joint work with: Sara Susca, Martínez

Scenario 1: translation into average consensus

- pseudo-distance between interpolation points (p_k, p_{k+1})

$$d(k) = \lambda \int_{p_k \to p_{k+1}} \kappa(\ell)^{1/3} d\ell + (1-\lambda) \int_{p_k \to p_{k+1}} d\ell$$

- "go to center of Voronoi cell" update is peer-to-peer averaging rule



- linear model is:
  - stochastic matrices: switching, symmetric and nondegenerate
  - union of associated graphs over time is a ring (i.e., jointly connected graphs)
  - convergence rate as in Toeplitz tridagonal problem

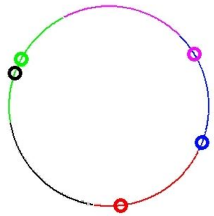
Scenario 2: Synchronized boundary patrolling

(i) some UAVs move along boundary of sensitive territory

(ii) short-range communication and sensing

(iii) surveillance objective:
minimize service time for appearing events
communication network connectivity

Example motion:



joint work with: Susca, Martínez

Analogy with mechanics and dynamics

(i) robots with "communication impacts" analogous to **beads on a ring**

(ii) classic subject in dynamical systems and geometric mechanics
billiards in polygons, iterated impact dynamics, gas theory of hard spheres

(iii) rich dynamics with even just 3 beads (distinct masses, elastic collisions)
dynamics akin billiard flow inside acute triangle
dense periodic and nonperiodic modes, chaotic collision sequences

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Iterated Impact Dynamics of *N*-Beads on a Ring*

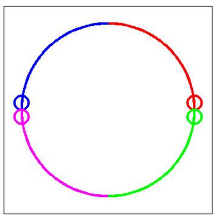
Bryan Cooley†
Paul K. Newton†

Abstract. When N-beads slide along a frictionless hoop, their collision sequence gives rise to a dynamical system that can be studied via matrix products. It is of general interest to understand the distribution of velocities and the corresponding eigenvalue spectrum that a given collision sequence can produce. We formulate the problem for general N and state some basic theorems regarding the eigenvalues of the collision matrices and their products. The

Boundary patrolling: synchronized bead oscillation

Desired synchronized behavior:

- starting from random initial positions and velocities
- every bead impacts its neighbor at the same point
- simultaneous impacts



Design specification for synchronization algorithm

Achieve: asymptotically stabilize synchronized motion
Subject to:

(i) arbitrary initial positions, velocities and directions of motion

(ii) beads can measure traveled distance, however
no absolute localization capability, no knowledge of circle length

(iii) no knowledge about n, adaptation to changing n (even and odd)

(iv) anynomous agents with memory and message sizes independent of n

(v) smooth dependency upon effect of measurement and control noise

Fully-adaptive feedback synchronization

Slowdown-Impact-Speedup algorithm

Algorithm: (for presentation's sake, beads sense their position)

1st phase: compute **average speed** v and **desired sweeping arcs**

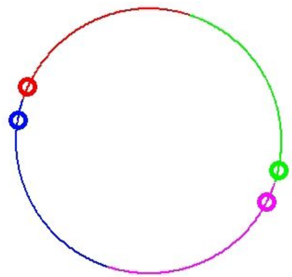
2nd phase for $f \in]\frac{1}{2}, 1[$, each bead:

- moves at nominal speed v if inside its desired sweeping arc
- slows down (fv) when moving away of its desired sweeping arc
  hesitate when early
- when impact, change direction
- speeds up when moving towards its desired sweeping arc

Simulations results: balanced synchronization

Balanced initial condition:

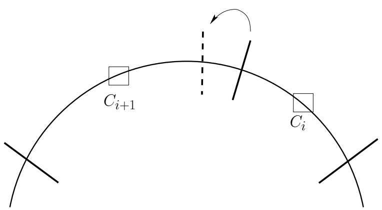
- n is even
- d_i is direction of motion
- $\sum_i^n d_i(0) = \sum_i^n d_i(t) = 0$
- $n/2$ move initially clockwise



First phase: average speed and sweeping arc

If an impact between bead i and $i+1$ occurs:

- beads average nominal speeds: $v_i^+ = v_{i+1}^+ = 0.5(v_i + v_{i+1})$
- beads change their direction of motion if $d_i = -d_{i+1}$ (head-head type)
- beads update their desired sweeping arc



exponential average consensus

Challenges

(i) how to prove balanced synchronization?

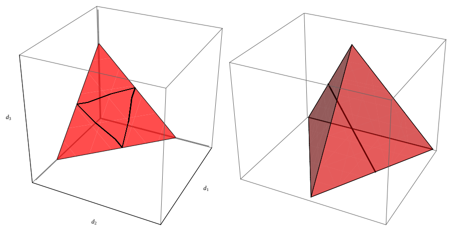
(ii) what happens for unbalanced initial conditions $\sum_i^n d_i(0) \neq 0$?

(iii) what happens for n is odd?

(iv) how to describe the system with a single variable?

Modeling detour

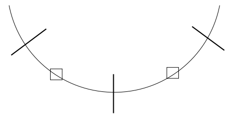
- configuration space
  (i) order-preserving dynamics $\theta_i \in \text{Arc}(\theta_{i-1}, \theta_{i+1})$ on $\mathbb{T}^n$
  (ii) $\Delta^n \times \{\text{c}, \text{cc}\}^n \times (\mathbb{R}_{>0})^n \times (\text{arcs})^n \times \{\text{away}, \text{towards}\}^n$



- hybrid system with
  (i) piecewise constant dynamics
  (ii) event-triggered jumps: impact, cross boundary

Passage and return times

- passage time: $t_i^k = k$th time when bead i passes by sweeping arc center



- return time: $\delta_i(t) =$ **duration between last two passage times**
- if impact between beads $(i, i+1)$ at time t, then

$$\begin{bmatrix} \delta_i \\ \delta_{i+1} \end{bmatrix}(t^+) = \underbrace{\begin{bmatrix} \frac{1-f}{1+f} & \frac{2f}{1+f} \\ \frac{2f}{1+f} & \frac{1-f}{1+f} \end{bmatrix}}_{\text{stochastic (irr + aperd)}} \begin{bmatrix} \delta_i \\ \delta_{i+1} \end{bmatrix}(t^-)$$

Convergence results: balanced synchronization

Balanced Synchronization Theorem: For balanced initial directions, assume

(i) exact average speed and desired sweeping arcs

(ii) initial conditions lead to well-defined 1st passage times

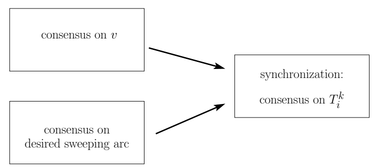
Then balanced synchronization is asymptotically stable

$$\lim_{t\to\infty} \delta(t) = \frac{2\pi}{Nv}\mathbf{1}_n, \qquad \lim_{k\to+\infty} \|T^k - \frac{\mathbf{1}_n \cdot T^k}{n}\mathbf{1}_n\| = 0$$

Conjectures arising from simulation results

Only assumption required is balanced initial conditions.

(i) analysis of cascade consensus algorithms

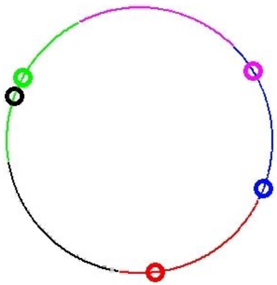


(ii) global attractivity of synchronous behavior

Simulations results: 1-unbalanced case

1-unbalanced initial condition:

- n is odd
- $\sum_i^n d_i(0) = \sum_i^n d_i(t) = \pm 1$



1-unbalanced synchronization

(i) $f \in]\frac{1}{2}, \frac{n}{1+n}[$

(ii) 1-unbalanced sync: beads meet at arcs boundaries $\pm \frac{2\pi}{n^2} \frac{f}{1-f}$

1-unbalanced Synchronization Theorem: For $\sum_i^n d_i(0) = \pm 1$, assume

(i) exact average speed and desired sweeping arcs

(ii) initial conditions lead to well-defined 1st passage times

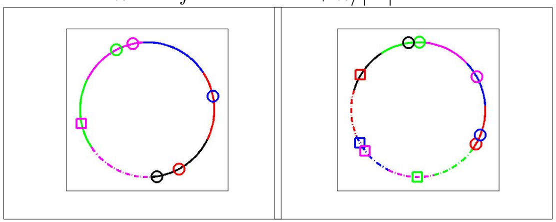
Then 1-unbalanced synchronization is asymptotically stable

$$\lim_{t\to\infty} \delta(t) = \frac{2\pi}{Nv} \mathbf{1}_n, \qquad \lim_{k\to+\infty} \left(T^{2k} - T^{2(k-1)}\right) = \frac{2}{v} \frac{2\pi}{n} \mathbf{1}_n$$

General unbalanced case

Conjecture global asy-synchronization in the balanced and unbalanced case

D-unbalanced period orbits Theorem:
Let $\sum_i^n d_i(0) = \pm D$. If there exists an orbit along which beads i and $i+1$ meet at boundary $\pm \frac{2\pi}{n^2} \frac{f}{1-f}$, then $f < \frac{n/|D|}{1+n/|D|}$.



Emerging discipline: motion-enabled networks

- **network modeling**
  network, ctrl+comm algorithm, task, complexity
- **coordination algorithm**
  deployment, task allocation, boundary estimation

Open problems

(i) algorithmic design for motion-enabled sensor networks
scalable, adaptive, asynchronous, agent arrival/departure
tasks: search, exploration, identify and track

(ii) integration between motion coordination, communication, and estimation tasks

(iii) Very few results available on:

(a) scalability analysis: time/energy/communication/control

(b) robotic networks over random geometric graphs

(c) complex sensing/actuation scenarios