Motion Coordination for Multi-Agent Networks

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AFOSR MURI Appers, ARO MURI Swarms, NSF Sensors, ONR YIP

Multi-agent networks

What kind of systems?

Groups of systems with control, sensing, communication and computing

Individual members in the group can

- sense its immediate environment
- communicate with others
- process the information gathered
- take a local action in response

Example networks from biology and engineering

Biological populations and swarms


Wildebeest herd in the Serengeti


Geese flying in formation


Atlantis aquarium, CDC Conference 2004

Multi-vehicle and sensor networks

embedded systems, distributed robotics

Distributed information systems, large-scale complex systems

intelligent buildings, stock market, self-managed air-traffic systems

Broad challenge

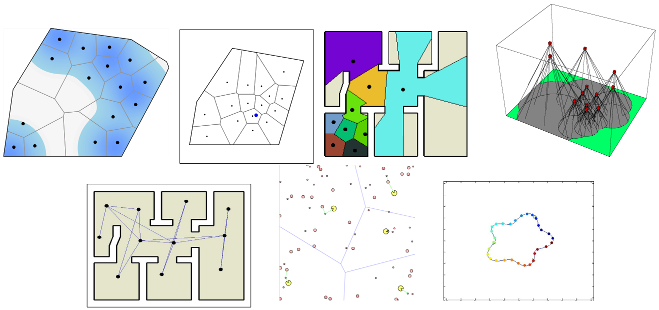
Useful engineering through small, inexpensive, limited-comm vehicles/sensors

Problem	lack of understanding of how to assemble and coordinate individual devices into a coherent whole
Distributed feedback	rather than "centralized computation for known and static environment"
Approach	integration of control, comm, sensing, computing

Research in Animation

(i) elementary motion tasks
deployment, rendezvous, flocking, self-assembly

(ii) sensing tasks
detection, localization, visibility, vehicle routing, search, plume tracing



Outline

I: Models for Multi-Agent/Robotic Networks: tools and modeling results

II: Motion Coordination: algorithms for multiple tasks
rendezvous, deployment

III: Sensing Tasks: sensing problems
target servicing, boundary estimation

Part I: Models for Multi-Agent Networks

References

(i) I. Suzuki and M. Yamashita. Distributed anonymous mobile robots: Formation of geometric patterns. *SIAM Journal on Computing*, 28(4):1347–1363, 1999

(ii) N. A. Lynch. *Distributed Algorithms*. Morgan Kaufmann Publishers, San Mateo, CA, 1997. ISBN 1558603484

(iii) D. P. Bertsekas and J. N. Tsitsiklis. *Parallel and Distributed Computation: Numerical Methods*. Athena Scientific, Belmont, MA, 1997. ISBN 1886529019

(iv) S. Martínez, F. Bullo, J. Cortés, and E. Frazzoli. On synchronous robotic networks – Part I: Models, tasks and complexity. *IEEE Transactions on Automatic Control*, April 2005. Submitted

Objective

(i) meaningful + tractable model

(ii) feasible operations and their cost

(iii) control/communication tradeoffs

Part I: Robotic network

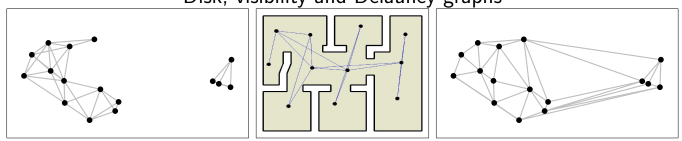
A **uniform/anonymous robotic network** $\mathcal{S}$ is

(i) $I = \{1, \dots, N\}$; **set of unique identifiers (UIDs)**

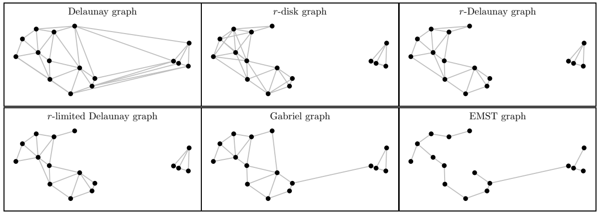
(ii) $\mathcal{A} = \{A_i\}_{i \in I}$, with $A_i = (X, U, X_0, f)$ is a set of identical control systems; **set of physical agents**

(iii) **interaction graph**

Disk, visibility and Delauney graphs



Communication models for robotic networks



Relevant graphs

(i) fixed, balanced

(ii) geometric or state-dependent

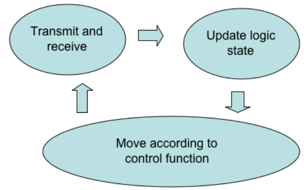
(iii) switching

(iv) random, random geometric

Message model: message, packet, bits; absolute or relative positions

Synchronous control and communication

(i) **communication schedule**	$\mathbb{T} = \{t_\ell\}_{\ell \in \mathbb{N}_0} \subset \overline{\mathbb{R}}_+$
(ii) **communication language**	L including the `null` message
(iii) set of values for **logic variables**	W
(iv) **message-generation function**	msg: $\mathbb{T} \times X \times W \times I \to L$
(v) **state-transition functions**	stf: $\mathbb{T} \times W \times L^N \to W$
(vi) **control function**	ctrl: $\overline{\mathbb{R}}_+ \times X \times W \times L^N \to U$



Task and complexity

- **Coordination task** is $(\mathcal{W}, \mathcal{T})$ where $\mathcal{T}\colon X^N \times \mathcal{W}^N \to \{\mathtt{true}, \mathtt{false}\}$

  Motion: deploy, gather, flock, reach pattern
  Logic-based: achieve consensus, synchronize, form a team
  Sensor-based: search, estimate, identify, track, map

- For $\{\mathcal{S}, \mathcal{T}, \mathcal{CC}\}$, define costs/complexity:
  control effort, communication packets, computational cost
- **Time complexity to achieve $\mathcal{T}$ with $\mathcal{CC}$**

$$\mathrm{TC}(\mathcal{T}, \mathcal{CC}, x_0, w_0) = \inf\{\ell \mid \mathcal{T}(x(t_k), w(t_k)) = \mathtt{true}\,, \text{ for all } k \geq \ell\}$$
$$\mathrm{TC}(\mathcal{T}, \mathcal{CC}) = \sup\left\{\mathrm{TC}(\mathcal{T}, \mathcal{CC}, x_0, w_0) \mid (x_0, w_0) \in X^N \times \mathcal{W}^N\right\}$$

Open problems in Part I

(i) complexity analysis (time/energy)

(ii) models/algorithms for asynchronous networks with agent arrival/departures

(iii) robotic network over random geometric graphs (multipath, fading)

(iv) parallel, sequential, hierarchical composition of behaviors

Part II: Motion Coordination

Scenarios examples of networks, tasks, ctrl+comm laws

(i) rendezvous
(ii) deployment

Rendezvous

(i) H. Ando, Y. Oasa, I. Suzuki, and M. Yamashita. Distributed memoryless point convergence algorithm for mobile robots with limited visibility. *IEEE Transactions on Robotics and Automation*, 15(5):818–828, 1999

(ii) J. Lin, A. S. Morse, and B. D. O. Anderson. The multi-agent rendezvous problem. In *IEEE Conf. on Decision and Control*, pages 1508–1513, Maui, HI, December 2003

(iii) J. Cortés, S. Martínez, and F. Bullo. Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions. *IEEE Transactions on Automatic Control*, 51(6), 2006. To appear

Deployment

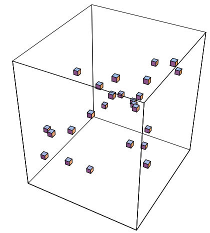
(i) J. Cortés, S. Martínez, T. Karatas, and F. Bullo. Coverage control for mobile sensing networks. *IEEE Transactions on Robotics and Automation*, 20(2):243–255, 2004

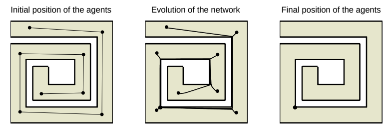
(ii) J. Cortés, S. Martínez, and F. Bullo. Spatially-distributed coverage optimization and control with limited-range interactions. *ESAIM. Control, Optimisation & Calculus of Variations*, 11:691–719, 2005

Scenario 1: aggregation laws for rendezous

Aggregation laws

```
At each comm round:
 1: acquire neighbors' positions
 2: compute connectivity constraint
    set
 3: move towards circumcenter of
    neighbors (while remaining
    connected)
```

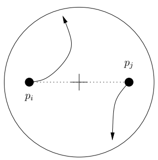


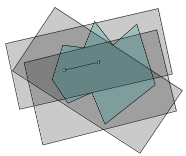


Task: rendezvous with connectivity constraint

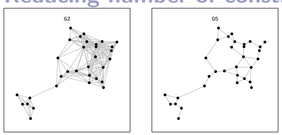
Scenario 1: aggregation laws for rendezous, cont'd

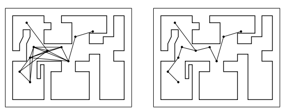
Pair-wise motion constraint set for connectivity maintenance





Reducing number of constraints





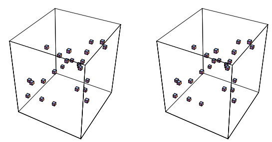
Scenario 1: Example complexity analysis

(i) first-order agents with disk graph, for $d = 1$,

$$\mathrm{TC}(\mathcal{T}_{\text{rendezvous}}, \mathcal{CC}_{\text{circumcenter}}) \in \Theta(N)$$

(ii) first-order agents with Delaunay graph, for $d = 1$,

$$\mathrm{TC}(\mathcal{T}_{(r\epsilon)\text{-rendezvous}}, \mathcal{CC}_{\text{circumcenter}}) \in \Theta(N^2 \log(N\epsilon^{-1}))$$



Example proof technique

For $N \geq 2$ and $a, b, c \in \mathbb{R}$, define the $N \times N$ Toeplitz matrices

$$\mathrm{Trid}_N(a,b,c) = \begin{bmatrix} b & c & 0 & \dots & 0 \\ a & b & c & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & a & b & c \\ 0 & \dots & 0 & a & b \end{bmatrix}$$

$$\mathrm{Circ}_N(a,b,c) = \mathrm{Trid}_N(a,b,c) + \begin{bmatrix} 0 & \dots & \dots & 0 & a \\ 0 & \dots & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ c & 0 & \dots & 0 & 0 \end{bmatrix}$$

To be studied for interesting a, b, c:
as stochastic matrices whose 2nd eigenvalue converges to 1 as $N \to +\infty$

Tridiagonal Toeplitz and circulant systems

Let $N \geq 2$, $\epsilon \in]0,1[$, and $a, b, c \in \mathbb{R}$. Let $x, y \colon \mathbb{N}_0 \to \mathbb{R}^N$ solve:

$$x(\ell+1) = \mathrm{Trid}_N(a,b,c)\, x(\ell), \qquad x(0) = x_0,$$
$$y(\ell+1) = \mathrm{Circ}_N(a,b,c)\, y(\ell), \qquad y(0) = y_0.$$

(i) if $a = c \neq 0$ and $|b| + 2|a| = 1$, then $\lim_{\ell \to +\infty} x(\ell) = \mathbf{0}$, and the maximum time required for $\|x(\ell)\|_2 \leq \epsilon \|x_0\|_2$ is $\Theta\big(N^2 \log \epsilon^{-1}\big)$;

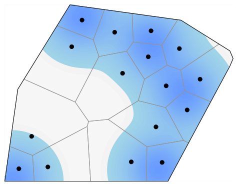
(ii) if $a \neq 0$, $c = 0$ and $0 < |b| < 1$, then $\lim_{\ell \to +\infty} x(\ell) = \mathbf{0}$, and the maximum time required for $\|x(\ell)\|_2 \leq \epsilon \|x_0\|_2$ is $O\big(N \log N + \log \epsilon^{-1}\big)$;

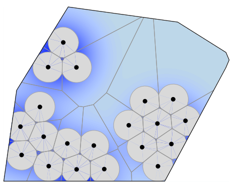
(iii) if $a \geq 0$, $c \geq 0$, $b > 0$, and $a + b + c = 1$, then $\lim_{\ell \to +\infty} y(\ell) = y_{\mathrm{ave}} \mathbf{1}$, where $y_{\mathrm{ave}} = \frac{1}{N} \mathbf{1}^T y_0$, and the maximum time required for $\|y(\ell) - y_{\mathrm{ave}} \mathbf{1}\|_2 \leq \epsilon \|y_0 - y_{\mathrm{ave}} \mathbf{1}\|_2$ is $\Theta\big(N^2 \log \epsilon^{-1}\big)$.

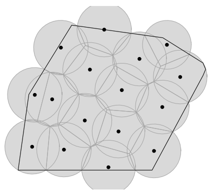
Scenario 2: dispersion laws for deployment

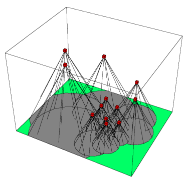
Dispersion laws

```
At each comm round:
 1: acquire neighbors' positions
 2: compute own dominance region
 3: move towards incenter /
    circumcenter / centroid of own
    dominance region
```









Scenarios: optimal deployment

ANALYSIS of cooperative distributed behaviors

(i) how do animals share territory?
what if every fish in a swarm goes
toward center of own dominance region?

Barlow, Hexagonal territories. Anim. Behav. '74

(ii) what if each vehicle moves toward center of mass of own Voronoi cell?

(iii) what if each vehicle moves away from closest vehicle?

DESIGN of performance metric

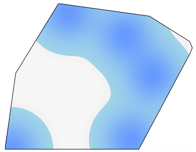
(iv) how to cover a region with n minimum radius overlapping disks?

(v) how to design a minimum-distortion (fixed-rate) vector quantizer? (Lloyd '57)

(vi) where to place mailboxes in a city / cache servers on the internet?

Scenario 2: general multi-center function

Objective: Given agents $(p_1, \dots, p_n)$ in convex environment Q
unspecified comm graph, achieve optimal coverage



Expected environment coverage

- let ϕ be distribution density function
- let f be a **performance/penalty function**

$f(\|q - p_i\|)$ is price for p_i to service q

- define **multi-center function**

$$\mathcal{H}_{\mathsf{C}}(p_1, \dots, p_n) = E_\phi \left[\min_i f(\|q - p_i\|) \right]$$

$$= \int_Q \min_i f(\|q - p_i\|)\phi(q)dq = \sum_i \int_{V_i} f(\|q - p_i\|)\phi(q)dq$$

Scenario 2: distributed gradient result

For a general non-decreasing $f : \overline{\mathbb{R}}_+ \to \mathbb{R}$
piecewise differentiable with finite-jump discontinuities at $R_1 < \dots < R_m$

Thm:

$$\frac{\partial \mathcal{H}_{\mathsf{C}}}{\partial p_i}(p_1, \dots, p_n) = \int_{V_i} \frac{\partial}{\partial p_i} f(\|q - p_i\|)\phi(q)dq$$

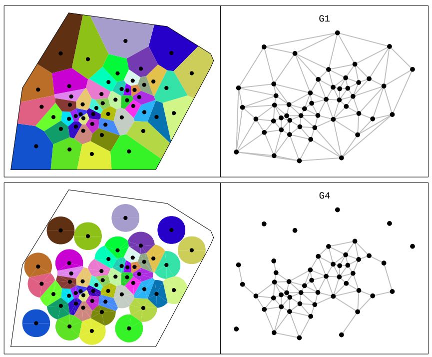
$$+ \sum_{\alpha=1}^{m} \Delta f_\alpha(R_\alpha) \Big(\sum_{k=1}^{M_i(2R_\alpha)} \int_{\mathrm{arc}_{i,k}(2R_\alpha)} n_{B_{R_\alpha}(p_i)} d\phi \Big)$$

$= \textbf{integral over } V_i \; + \; \textbf{integral along arcs inside } V_i$

Gradient depends on information contained in V_i

On Voronoi and limited-range Voronoi partitions

Problem: $\frac{\partial \mathcal{H}_{\mathsf{C}}}{\partial p_i}$ is distributed over **Delaunay graph**, but not **disk graph**
Solution: modify function so that its gradient is distributed over **disk graph**



Scenario 2: truncation

problem $\partial \mathcal{H}_{\mathsf{C}}$ distributed over Delaunay graph, but comm. is r-disk graph

approach truncate $f_{\frac{r}{2}}(x) = f(x)\; 1_{[0,\frac{r}{2})}(x) + (\sup_Q f) \cdot 1_{[\frac{r}{2},+\infty)}(x)$,

$$\mathcal{H}_{\frac{r}{2}}(p_1, \dots, p_n) = E_\phi \left[\min_i f_{\frac{r}{2}}(\|q - p_i\|) \right]$$

Result 1: Gradient of $\mathcal{H}_{\frac{r}{2}}$ is **distributed over limited-range Delaunay**

$$\frac{\partial \mathcal{H}_{\frac{r}{2}}}{\partial p_i} = \textbf{integral over } V_i \cap B_{\frac{r}{2}}(p_i) + \textbf{integral along arcs inside } V_i \cap B_{\frac{r}{2}}(p_i)$$

Result 2: $\mathcal{H}_{\mathsf{C}}$ constant-factor approximation

$$\beta\, \mathcal{H}_{\frac{r}{2}}(P) \leq \mathcal{H}_{\mathsf{C}}(P) \leq \mathcal{H}_{\frac{r}{2}}(P), \quad \beta = \left(\frac{r}{2\,\mathrm{diam}(Q)}\right)^2$$

Aggregate objective functions

design of aggregate network-wide cost/objective/utility functions

- objective functions to encode motion coordination objective
- objective functions as Lyapunov functions
- objective functions for gradient flows

	$\mathcal{H}_{\mathrm{C}}$	$\mathcal{H}_{\mathrm{area}}$	$\mathcal{H}_{\mathrm{diam}}$
DEFINITION	$E\left[\min d(q, p_i)\right]$	$\text{area}_\phi(\cup_i B_{r/2}(p_i))$	$\max\limits_{i,j} \Vert p_i - p_j \Vert$
SMOOTHNESS	C^1	globally Lipschitz	continuous, locally Lipschitz
CRITICAL POINTS MINIMA	Centroidal Voronoi configurations	r-limited Voronoi configurations*	common location for p_i
HEURISTIC DESCRIPTION	expected distortion	area covered	diameter connected component

Open problems in Part II

(i) general pattern formation problem

(ii) static and dynamic motion patterns

(iii) algorithms for line-of-sight 3D networks

(iv) connectivity and collision avoidance algorithms

Part III on Sensing Tasks

Problems of interest

- optimal sensor placement
- localization, estimation
- distributed sensing tasks:
  search, exploration, map building, target identification

References on Target Servicing

(i) R. W. Beard, T. W. McLain, M. A. Goodrich, and E. P. Anderson. Coordinated target assignment and intercept for unmanned air vehicles. *IEEE Transactions on Robotics and Automation*, 18(6):911–922, 2002

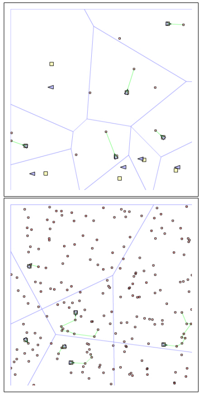
(ii) A. E. Gil, K. M. Passino, and A. Sparks. Cooperative scheduling of tasks for networked uninhabted autonomous vehicles. In *IEEE Conf. on Decision and Control*, pages 522–527, Maui, Hawaii, December 2003

(iii) W. Li and C. G. Cassandras. Stability properties of a cooperative receding horizon controller. In *IEEE Conf. on Decision and Control*, pages 492–497, Maui, HI, December 2003

(iv) E. Frazzoli and F. Bullo. Decentralized algorithms for vehicle routing in a stochastic time-varying environment. In *IEEE Conf. on Decision and Control*, pages 3357–3363, Paradise Island, Bahamas, December 2004

Scenario 3: Vehicle Routing

Objective: Given agents $(p_1, \ldots, p_n)$ moving in environment Q
service targets in environment



Model:

- targets arise randomly in space/time
- vehicle know of targets arrivals
- low and high traffic scenarios

Scenario 3 service targets quickly

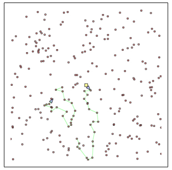
Scenario 3: receding-horizon TSP algorithm, I

Name: (Single Vehicle) Receding-horizon TSP

For $\eta \in (0, 1]$, single agent performs:

1: while no targets, dispersion/coverage algorithm ($f(x) = x$)
2: while targets waiting
   (i) compute optimal TSP tour through all targets
   (ii) service the η-fraction of tour with maximal number of targets

Asymptotically constant-factor optimal in light and high traffic



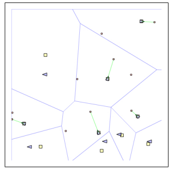
Scenario 3: receding-horizon TSP algorithm, II

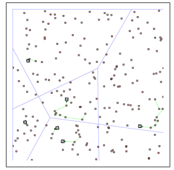
Name: Receding-horizon TSP

For $\eta \in (0, 1]$, agent i performs:

1: compute own Voronoi cell V_i
2: apply Single-Vehicle RH-TSP policy on V_i

Asymptotically constant-factor optimal in light and high traffic (simulations only)





Emerging Motion Coordination Discipline

(i) **network modeling**
network, ctrl+comm algorithm, task, complexity

coordination algorithm
optimal deployment, rendezvous, vehicle routing
scalable, adaptive, asynchronous, agent arrival/departure

(ii) **Systematic algorithm design**

- meaningful aggregate cost functions
- class of (gradient) algorithms local, distributed
- geometric graphs
- stability theory for networked hybrid systems

Motion Coordination for Visually-guided Agents

25th Benelux Meeting on Systems and Control
Heeze, The Netherlands, March 13-15, 2006

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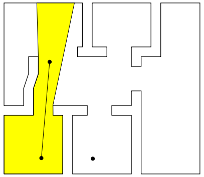
Visually-guided agents

- Environment
  Polygon, Q: non self-intersecting with well-defined interior and exterior
- Visibility

  Visibility polygon $S(p) = \{q \mid q \text{ is visible from } p\}$



- Sensing and communication within visibility polygon
- Visually-guided agent
  Point robot with omnidirectional vision
  First order dynamics: $p(k+1) = p(k) + u$, $\dot{p} = u$

Deployment and Rendezvous

- **Deployment**
  Cover a given environment
  Objective: every point is visible to at least one sensor
- **Rendezvous**
  Gather a previously deployed network at one location

References

(i) A. Ganguli, J. Cortés, and F. Bullo. Maximizing visibility in nonconvex polygons: Nonsmooth analysis and gradient algorithm design. *SIAM Journal on Control and Optimization*, March 2006a. To appear

(ii) A. Ganguli, J. Cortés, and F. Bullo. Distributed deployment of asynchronous guards in art galleries. In *American Control Conference*, Minneapolis, MN, June 2006b. To appear
A. Ganguli, J. Cortés, and F. Bullo. Deployment of connected network of guards in art galleries. In *IEEE Conf. on Decision and Control*, San Diego, CA, December 2006c. Submitted

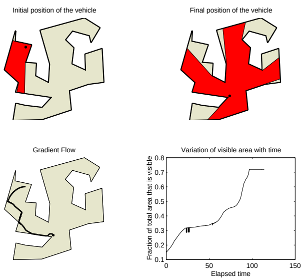
(iii) A. Ganguli, J. Cortés, and F. Bullo. On rendezvous for visually-guided agents in a nonconvex polygon. In *IEEE Conf. on Decision and Control*, pages 5686–5691, Seville, Spain, December 2005

R1: Visibility-based deployment of a single agent

Problem: Design continuous time algorithm to increase visible area

Motivation:

- Optimal sensor placement problem
- Next Best View problem in robotics



Approach: Gradient flow:

1: compute visibility polygon $S(p(t))$
2: compute gradient of $A \circ S(p(t))$
3: take a step in gradient direction

R2: Visibility-based deployment of multiple agents

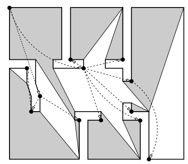
Problem: Achieve complete visibility of nonconvex environment with line-of-sight interactions and asynchronous operation

Motivation:

- Surveillance, map building, search

Approach:

1: Partition environment into star-shaped polygons
2: Cover the nodes of **dual graph** by:
   - Node-to-node and global navigation
   - Dispersing by comparing UIDs
   - Distributed information processing

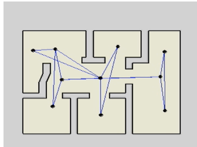


R3: Rendezvous of visually-guided agents

Problem: Gather all agents at a single location
with line-of-sight sensing and no communication

Motivation:

- Basic task in multi-vehicle networks
- Collection of sensors after completion of a task



Approach: at all times, each agent

1. computes positions of all other visible agents
2. construct motion contraint set
3. moves “closer” while maintaining connectivity

Outline

(i) Visibility-based deployment of a single agent

(ii) Visibility-based deployment of multiple agents

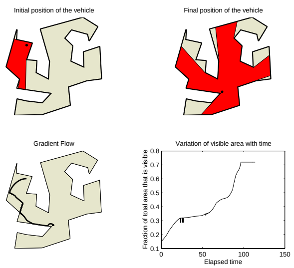
(iii) Rendezvous of visually-guided agents

R1: Visibility-based deployment of a single agent

Problem: Design continuous time algorithm to increase visible area

Motivation:

- Optimal sensor placement problem
- Next Best View problem in robotics



Approach: Gradient flow:

1. compute visibility polygon $S(p(t))$
2. compute gradient of $A \circ S(p(t))$
3. take a step in gradient direction

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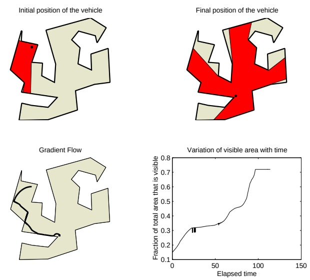


Approach: Gradient flow:

1. compute visibility polygon $S(p(t))$
2. compute gradient of $A \circ S(p(t))$
3. take a step in gradient direction

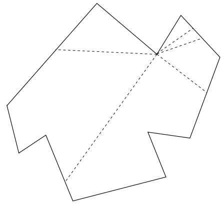
Approach

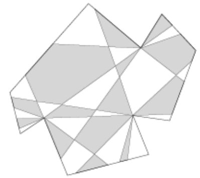
- Characterize the objective
  Maximize the area of the visibility polygon function, $A \circ S : Q \mapsto \mathbb{R}_+$
- Algorithm design
  Gradient-based
- Convergence analysis
  Use some form of LaSalle Invariance Principle with $A \circ S$ as Lyapunov function

Area of visibility polygon

Results on smoothness of $A \circ S : Q \mapsto \mathbb{R}_+$

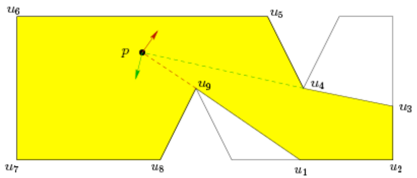
- Discontinuous at reflex vertices, but locally Lipschitz everywhere else

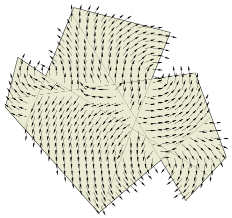




- Differentiable away from **generalized inflection segments**
- There exist polygons where $A \circ S$ and $-A \circ S$ are not regular everywhere

Gradient of the area of visibility polygon





- Away from boundary and generalized inflection segments

$$\frac{\partial}{\partial p} A \circ S(p) = \sum_{i=1}^{k} \frac{\partial A(u_1, \dots, u_k)}{\partial u_i} \frac{\partial u_i}{\partial p}(p)$$

$$\frac{\partial u_i}{\partial p}(p) \cdot \dot{p} = \frac{\mathrm{dist}(v_i, \ell)}{(\mathrm{dist}(p, \ell) - \mathrm{dist}(v_i, \ell))^2} \big(\text{perp. to visibility } \cdot \dot{p}\big) \big(\text{versor along } \ell\big)$$

- smooth boundaries and 3D environments

Main results

- Almost everywhere differential equation for observer

$$\dot{p}(t) = X_Q(p(t))$$

- Differential inclusion and Filippov solutions

Theorem
Any solution $\gamma \colon \overline{\mathbb{R}}_+ \to Q$ of X_Q has the following properties:
(i) $A \circ S(\gamma)$ is regular almost everywhere
(ii) $t \mapsto A \circ S(\gamma(t))$ is continuous and monotonically nondecreasing
(iii) γ approaches $\{\text{critical points of } A \circ S\} \cup$ reflex vertices

Nonsmooth LaSalle Invariance Principle

Existing Literature

- V is C^1 and $\dot{V} \leq 0$ (LaSalle '68)
- V is locally Lipschitz and regular (Bacciotti and Ceragioli '99)
- V is locally Lipschitz (Ryan '98)

Today

- V is locally Lipschitz and $V \circ \gamma$ is regular a.e.
- V is locally Lipschitz everywhere except at a finite number of points, and $V \circ \gamma$ is regular a.e.

Theorem Let $C \subset S$ be finite, and $V : S \to \mathbb{R}$ be locally Lipschitz on $S \setminus C$, bounded from below on S. Assume:

(A1) set-valued Lie derivative is negative semi-definite

(A2) if γ is a Filippov solution with $\gamma(0) \in C$, then $\lim_{t \to 0^-} V(\gamma(t)) \geq \lim_{t \to 0^+} V(\gamma(t))$

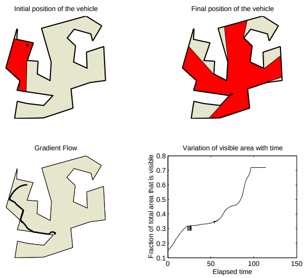
(A3) if $\gamma : \overline{\mathbb{R}}_+ \to S$ is a Filippov solution of X, then $V \circ \gamma$ is regular almost everywhere.

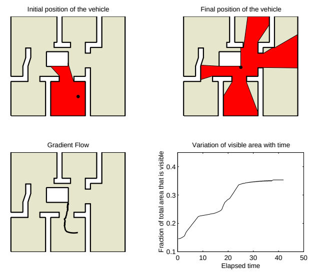
Then

each Filippov solution of X with initial condition in S approaches as $t \to +\infty$

largest weakly invariant set in $\left(\overline{\left\{x \in S \setminus C \mid 0 \in \widetilde{\mathcal{L}}_X V(x)\right\}} \cup C\right)$

Simulation results





Outline

(i) Visibility-based deployment of a single agent

- First provably correct algorithm for this version of *Next Best View* problem
- General results on nonsmooth analysis and control design
- Simulations show that in the presence of noise a local maximum is reached

(ii) **Visibility-based deployment of multiple agents**

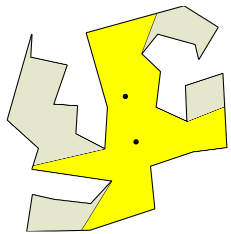
(iii) Rendezvous of visually-guided agents

Multi-agent gradient ascent

Assume network is connected

Each agent performs the following actions:

1: Compute visibility polygon $S(p_i)$
2: Wait for N communication hops to compute $V = \cup_i^N S(p_i)$
3: Compute $\frac{\partial V}{\partial p_i}$
4: Take a step in the direction of gradient

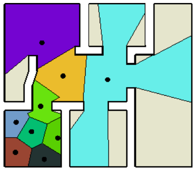


Gradient-based approach

Partition-based approach

Each agent performs the following actions:

1. Compute visibility polygon $S(p_i)$
2. Compute the set of points, $C(p_i)$ in $S(p_i)$ for which either p_i is the only agent within line-of-sight or the nearest
3. Take the connected component of $C(p_i)$ containing p_i
4. Move toward the furthest point in this set



Partition-based approach

R2: Visibility-based deployment of multiple agents

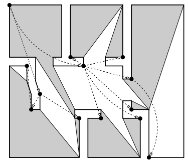
Problem: Achieve complete visibility of nonconvex environment with line-of-sight interactions and asynchronous operation

Motivation:

- Surveillance, map building, search

Approach:

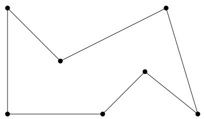
1. Partition environment into star-shaped polygons
2. Cover the nodes of **dual graph** by:
   - Node-to-node and global navigation
   - Dispersing by comparing UIDs
   - Distributed information processing



Art Gallery Problem and Theorem

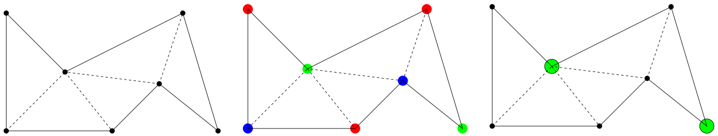
Art Gallery Problem (Klee '73):

Imagine placing guards inside a nonconvex polygon with n vertices: how many guards are required and where should they be placed in order for each point in the polygon to be visible by at least one guard?



Theorem (Chvátal '75): $\lfloor n/3 \rfloor$ guards are sufficient and sometimes necessary

"Triangulation + coloring" proof (Fisk '78):

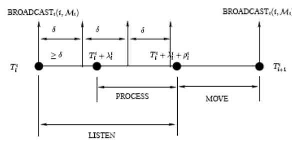


Network model

Specifications

- Sensing region: $S(p)$
- Comm. region: $S(p) \cap B(p, r)$, $r \leq R$
- Each agent has UID, i, position p_i
- $\mathcal{M}_i$ denotes memory contents
- BROADCAST$_i(i, \mathcal{M}_i)$ denotes broadcast containing UID and memory
- RECEIVE$_i(j, \mathcal{M}_j)$ denotes broadcast from agent j

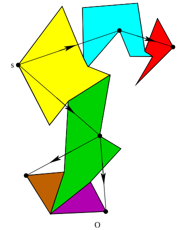
Bounded delay δ between BROADCAST and corresponding RECEIVE



Approach

(i) **Represent the environment by a graph**
(ii) Node-to-node navigation and deployment over a graph
(iii) Distributed information exchange

Vertex-induced tree

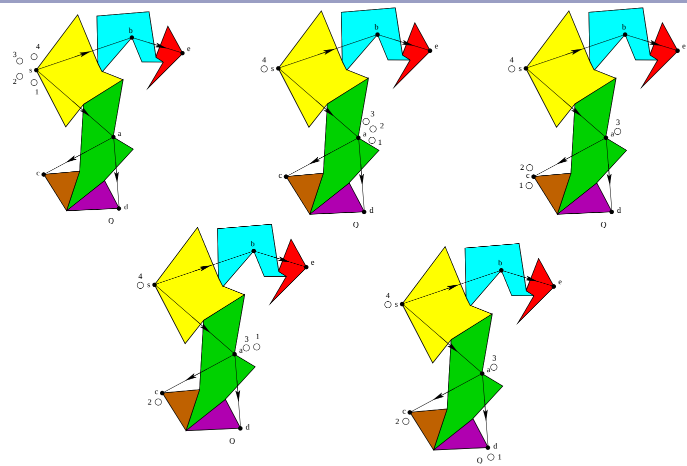


- the graph $\mathcal{G}_Q(s)$ is a rooted tree
- no two nodes sharing an edge are visible to each other
- maximum # nodes in the vertex-induced tree is $\lfloor \frac{n}{2} \rfloor$, where $n = |\,\mathrm{Ve}(Q)|$

Approach

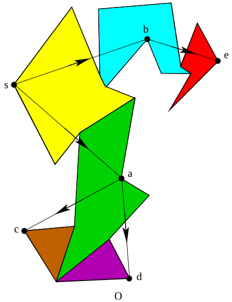
(i) Represent the environment by a graph
(ii) **Node-to-node navigation and deployment over a graph**
(iii) Distributed information exchange

Depth-first deployment



Randomized deployment

Compare ID's, and
Perform random search



Example: $P[s|a] = P[c|a] = P[d|a] = \frac{1}{3}$

Deployment over graph algorithms

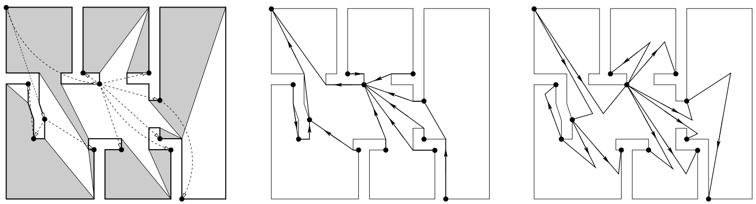
Assume: All agents initially at root s

Agent i performs

compares UID with agents at the same node
if i is largest UID **then**
    stay
else
    obtain $\mathcal{M}$ from agent with maximum UID
    move according to depth-first or randomized deployment
end if

Node-to-node navigation

the planned paths "from node to parent" and "from node to children:"



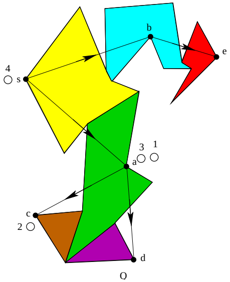
Approach

(i) Represent the environment by a graph

(ii) Node-to-node navigation and deployment over a graph

(iii) **Distributed information exchange**

Geographic information required for navigation



- Required memory: $\mathcal{M} = \{p_{\text{parent}}, p_{\text{last}}, v', v''\}$
  p_{parent} is parent node to current agent's position
  p_{last} is last node visited by the agent
  (v', v'') is the gap toward the parent node
- Init: four values set to the initial agent position
- Actions:
  (i) $\mathcal{M}$ broadcast together with UID during the SPEAK
  (ii) After move from k_{parent} to k_{child} through gap g_1, g_2, update: $p_{\text{parent}} := k_{\text{parent}}$, $p_{\text{last}} := k_{\text{parent}}$, $(v', v'') := (g_1, g_2)$
  (iii) After move from k_{child} to k_{parent}, update: $p_{\text{last}} := k_{\text{child}}$ and agent acquires correct $\{p_{\text{parent}}, v', v''\}$ from incoming messages

Main results

Theorem (Depth-first deployment)

(i) In finite time t^* there will be at least one agent on $\min\{N, |\mathcal{G}_Q(s)|\}$ nodes of $\mathcal{G}_Q(s)$.

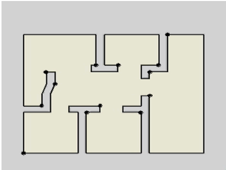
(ii) If there exist bounds $\lambda_{\max}$ and $\rho_{\max}$ such that $\lambda_l^i \leq \lambda_{\max}$ and $\rho_l^i \leq \rho_{\max}$ for all i and l, then

$$t^* \leq \mathcal{T}_{\text{motion}} + \mathcal{T}_{\text{comm/comp}},$$

where $\mathcal{T}_{\text{motion}} \leq 2\Big(\mathcal{L}_{\text{ford}}(\mathcal{G}_Q(s)) + \mathcal{L}_{\text{back}}(\mathcal{G}_Q(s))\Big)$ and $\mathcal{T}_{\text{comm/comp}} \leq 2(m-1)(\lambda_{\max} + \rho_{\max})$. Also,

$$t^* \in \Theta(N).$$

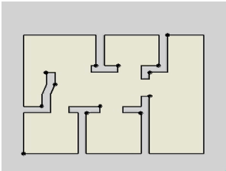
(iii) If $N \geq \frac{n}{2}$, then visibility-based deployment achieved at t^*.



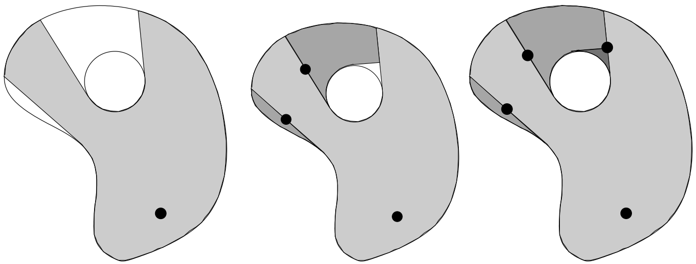
Theorem (Randomized deployment)

(i) in finite time with high probability there is at least one agent on $\min\{|\mathcal{N}_Q(s)|, N\}$ nodes of $\mathcal{G}_Q(s)$

(ii) if $N \geq \frac{n}{2}$, then the visibility-based deployment problem is solved in finite time with high probability



Connected deployment in orthogonal galleries



- general partition algorithms
- connectivity of visibility graph

Outline

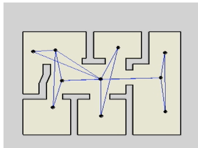
(i) Visibility-based deployment of a single agent
(ii) Visibility-based deployment of multiple agents
  (a) Visibility-based deployment solved when number of agents is at least $\frac{n}{2}$
  (b) Asynchronous setting
  (c) Time complexity investigated
(iii) **Rendezvous of visually-guided agents**

R3: Rendezvous of visually-guided agents

Problem: Gather all agents at a single location
with line-of-sight sensing and no communication

Motivation:

- Basic task in multi-vehicle networks
- Collection of sensors after completion of a task

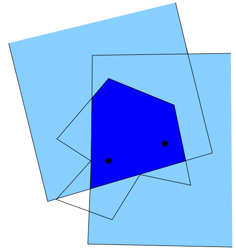


Approach: at all times, each agent

1. computes positions of all other visible agents
2. construct motion contraint set
3. moves "closer" while maintaining connectivity

Preserving visibility

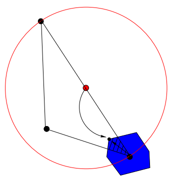
Build convex constraint sets for every visible pair



Sets change continuously as the position of the points
Sets are "large enough"

Move closer: Circumcenter algorithm

Each agent moves towards the circumcenter of set comprising of neighbors and itself



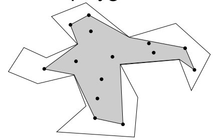
Each agent p_i executes the following at each time instant:

1. acquire set of neighbors, $\mathcal{N}_i$
2. compute intersection of constraint sets, $\mathcal{C}_i$
3. compute intersection of $\mathcal{C}_i$ with convex hull of $\mathcal{N}_i \cup \{p_i\}$, X_i
4. compute circumcenter of $\mathcal{N}_i \cup \{p_i\}$, CC_i
5. move toward CC_i remaining inside X_i

Analysis

LaSalle Invariance Principle for set-valued maps

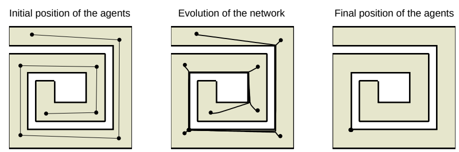
Lyapunov function
minimum perimeter of enclosing polygons



Smoothness of algorithm
Circumcenter algorithm T_G is continuous if $G = \mathcal{G}_{\text{vis},Q}$ is fixed
Define set-valued map $T\colon Q^n \to 2^{(Q^n)}$

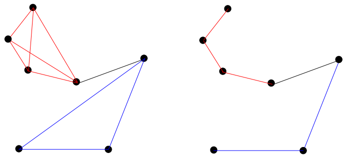
$$T(P) = \{T_{\mathcal{G}}(P) \in Q^n \mid \mathcal{G} \text{ is connected}\}$$

Key fact: T is upper semi-continuous

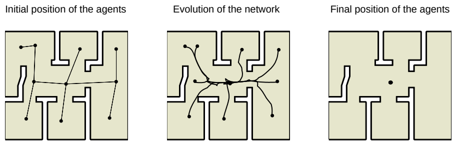


Reduction in connectivity constraints

Objective
Reduce connectivity constraints while preserving connected components
Distributed computation



Applicable to any graph where a node can detect a clique if it is present in it



Summary of "Visual Coordination"

Algorithms for elementary tasks

(i) Optimal location a single agent
(ii) A distributed version of the Art Gallery Problem
(iii) The rendezvous problem

Emerging Motion Coordination Discipline

- **network modeling**
  network, ctrl+comm algorithm, task, complexity
- **coordination algorithm**
  optimal deployment, rendezvous, vehicle routing
  scalable, adaptive, asynchronous, agent arrival/departure
- **Systematic algorithm design**

(i) geometric graphs
(ii) meaningful aggregate cost functions
(iii) class of (gradient) algorithms local, distributed
(iv) distributed information processing
(v) stability theory for networked hybrid systems